UNIT 3

Module 8

Functional Relationships

COMMON CORE GPS

Contents

MATHEMATICAL PRACTICES The Common Core Georgia Performance Standards for Mathematical Practice describe varieties of expertise that all students should seek to develop. Opportunities to develop these practices are integrated throughout this program.

1 Make sense of problems and persevere in solving them.
2 Reason abstractly and quantitatively.
3 Construct viable arguments and critique the reasoning of others.
4 Model with mathematics.
5 Use appropriate tools strategically.
6 Attend to precision.
7 Look for and make use of structure.
8 Look for and express regularity in repeated reasoning.

Unpacking the Standards

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Multilingual Glossary

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this chapter.

COMMON CORE GPS **MCC9-12.F.IF.1**

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. …

Key Vocabulary

function (función)
A relation in which every domain value is paired with exactly one range value.

domain (dominio)
The set of all first coordinates (or *x*-values) of a relation or function.

range of a function or relation (rango de una función o relación)
The set of all second coordinates (or *y*-values) of a function or relation.

element (elemento)
Each member in a set.

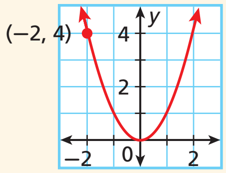
What It Means For You

A function model guarantees you that for any input value, you will get a unique output value.

EXAMPLE **Relationship is a function**

$y = x^2$

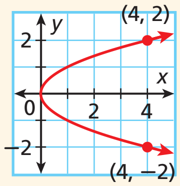
One output for every input: When −2 is input, the output is always 4.



NON-EXAMPLE **Relationship is NOT a function**

$y^2 = x$

Two outputs for every input but 0: When 4 is input, the output can be −2 or 2.



COMMON CORE GPS **MCC9-12.F.IF.4**

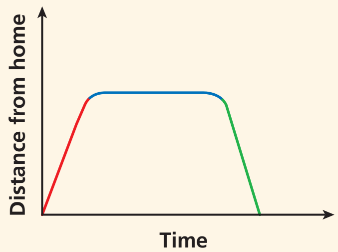
For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

What It Means For You

Learning to interpret a graph enables a deep visual understanding of all sorts of relationships.

EXAMPLE

A group of friends walked to the town market, did some shopping there, then returned home.



8-1 Graphing Relationships

Essential Question: How can you use key features to sketch a graph of a real-world situation?

Objectives

Match simple graphs with situations.

Graph a relationship.

Vocabulary

continuous graph
discrete graph

Who uses this?

Cardiologists can use graphs to analyze their patients' heartbeats. (See Example 2.)

Graphs can be used to illustrate many different situations. For example, trends shown on a cardiograph can help a doctor see how the patient's heart is functioning.

To relate a graph to a given situation, use key words in the description.

COMMON CORE GPS **EXAMPLE 1** MCC9-12.F.IF.4

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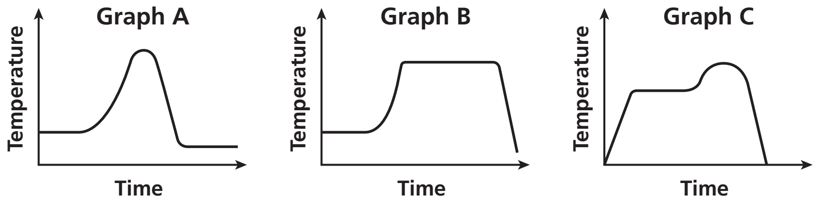
Relating Graphs to Situations

The air temperature was constant for several hours at the beginning of the day and then rose steadily for several hours. It stayed the same temperature for most of the day before dropping sharply at sundown. Choose the graph that best represents this situation.



Step 1 Read the graphs from left to right to show time passing.

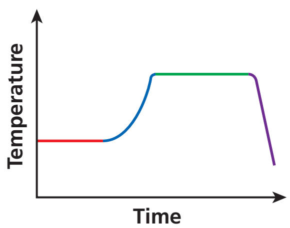
Step 2 List key words in order and decide which graph shows them.

Key Words	Segment Description	Graphs
Was constant	Horizontal	Graphs A and B
Rose steadily	Slanting upward	Graphs A and B
Stayed the same	Horizontal	Graph B
Dropped sharply	Slanting downward	Graph B

Step 3 Pick the graph that shows all the key phrases in order.

horizontal, slanting upward, horizontal, slanting downward

The correct graph is B.

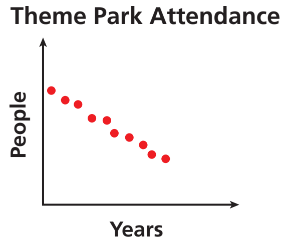


CHECK IT OUT!

1. The air temperature increased steadily for several hours and then remained constant. At the end of the day, the temperature increased slightly again before dropping sharply. Choose the graph above that best represents this situation.

As seen in Example 1, some graphs are connected lines or curves called **continuous graphs**. Some graphs are only distinct points. These are called **discrete graphs**.

The graph on theme-park attendance is an example of a discrete graph. It consists of distinct points because each year is distinct and people are counted in whole numbers only. The values between the whole numbers are not included, since they have no meaning for the situation.



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EXAMPLE 2 MCC9-12.F.IF.4

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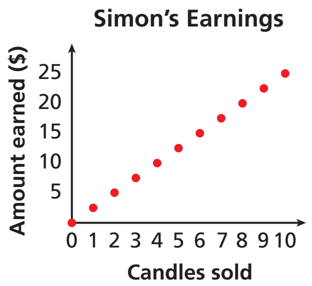
Sketching Graphs for Situations

Sketch a graph for each situation. Tell whether the graph is continuous or discrete.

A **Simon is selling candles to raise money for the school dance. For each candle he sells, the school will get $2.50. He has 10 candles that he can sell.**



The amount earned (y-axis) increases by $2.50 for each candle Simon sells (x-axis).

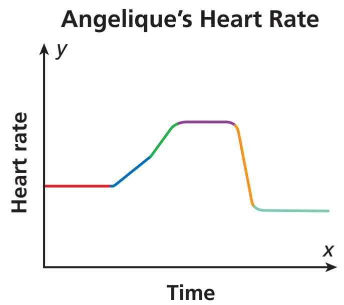
Since Simon can only sell whole numbers of candles, the graph is 11 distinct points.

The graph is discrete.

B **Angelique's heart rate is being monitored while she exercises on a treadmill. While walking, her heart rate remains the same. As she increases her pace, her heart rate rises at a steady rate. When she begins to run, her heart rate increases more rapidly and then remains high while she runs. As she decreases her pace, her heart rate slows down and returns to her normal rate.**

As time passes during her workout (moving left to right along the x-axis), her heart rate (y-axis) does the following:

- **remains the same**,
- **rises at a steady rate**,
- **increases more rapidly** (**steeper** than previous segment),
- **remains high**,
- **slows down**,
- and then returns to her **normal rate**.



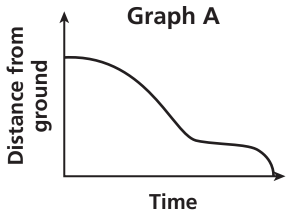
The graph is continuous.

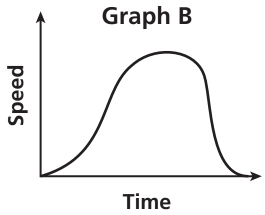
CHECK IT OUT! **Sketch a graph for each situation. Tell whether the graph is continuous or discrete.**

2a. Jamie is taking an 8-week keyboarding class. At the end of each week, she takes a test to find the number of words she can type per minute. She improves each week.

2b. Henry begins to drain a water tank by opening a valve. Then he opens another valve. Then he closes the first valve. He leaves the second valve open until the tank is empty.

When sketching or interpreting a graph, pay close attention to the labels on each axis. Both graphs below show a relationship about a child going down a slide. **Graph A** represents the child's *distance from the ground* over time. **Graph B** represents the child's *speed* over time.





COMMON CORE GPS **EXAMPLE 3** Ext. of MCC9-12.F.IF.4

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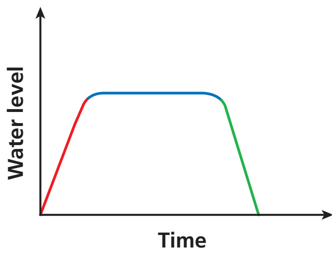
Writing Situations for Graphs

Write a possible situation for the given graph.



Step 1 Identify labels.

x-axis: time y-axis: water level

Step 2 Analyze sections.

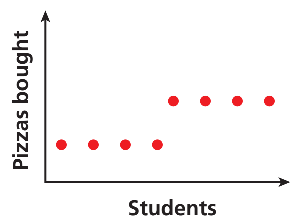
Over time, the water level

- increases steadily,
- remains unchanged,
- and then decreases steadily.

Possible Situation: A watering can is filled with water. It sits for a while until some flowers are planted. The water is then emptied on top of the planted flowers.

CHECK IT OUT!

3. Write a possible situation for the given graph.



MCC.MP.2, MCC.MP.4 MATHEMATICAL PRACTICES

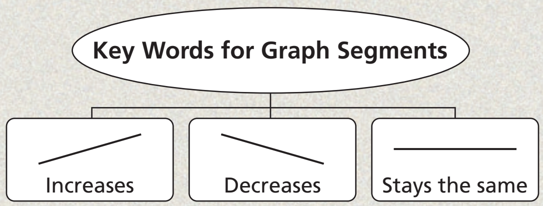
THINK AND DISCUSS

1. Should a graph of age related to height be a continuous graph or a discrete graph? Explain.

2. Give an example of a situation that, when graphed, would include a horizontal segment.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. Write an example of key words that suggest the given segments on a graph. One example for each segment is given for you.



8-1 Exercises

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Homework Help

GUIDED PRACTICE

Vocabulary **Apply the vocabulary from this lesson to answer each question.**

1. A ____?____ graph is made of connected lines or curves. (*continuous* or *discrete*)

2. A ____?____ graph is made of only distinct points. (*continuous* or *discrete*)

SEE EXAMPLE 1

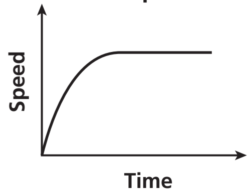
Choose the graph that best represents each situation.

3. A person alternates between running and walking.

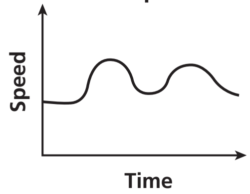
4. A person gradually speeds up to a constant running pace.

5. A person walks, gradually speeds up to a run, and then slows back down to a walk.

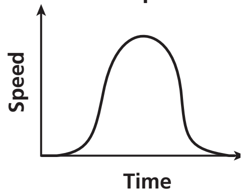
Graph A



Graph B



Graph C



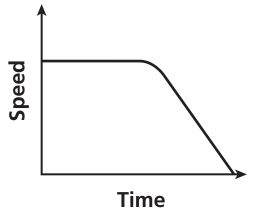
SEE EXAMPLE 2

6. Maxine is buying extra pages for her photo album. Each page holds exactly 8 photos. Sketch a graph to show the maximum number of photos she can add to her album if she buys 1, 2, 3, or 4 extra pages. Tell whether the graph is continuous or discrete.

SEE EXAMPLE 3

Write a possible situation for each graph.

7.



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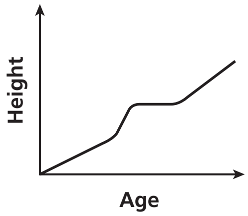

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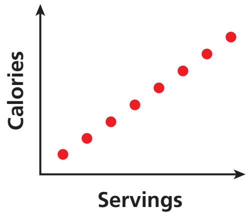

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
10–12	1
13	2
14–16	3

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Online Extra Practice

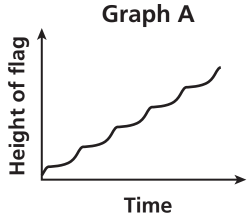
Choose the graph that best represents each situation.

10. A flag is raised up a flagpole quickly at the beginning and then more slowly near the top.

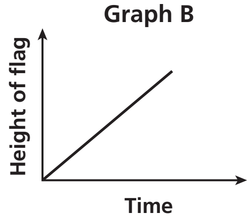
11. A flag is raised up a flagpole in a jerky motion, using a hand-over-hand method.

12. A flag is raised up a flagpole at a constant rate of speed.

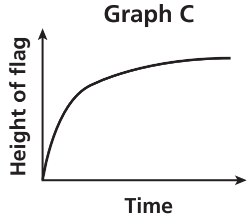
Graph A



Graph B



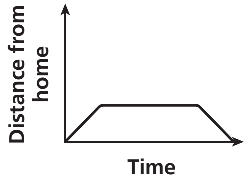
Graph C



13. For six months, a puppy gained weight at a steady rate. Sketch a graph to illustrate the weight of the puppy during that time period. Tell whether the graph is continuous or discrete.

Write a possible situation for each graph.

14.



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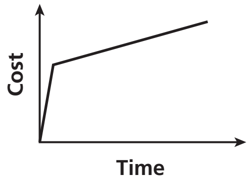
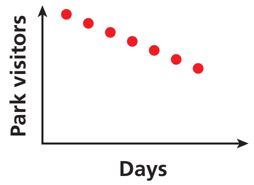


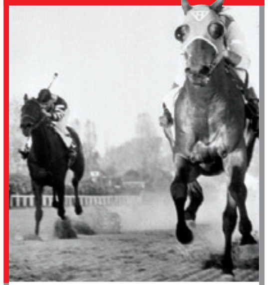
16.



17. **Data Collection** Use a graphing calculator and motion detector for the following.
   a. On a coordinate plane, draw a graph relating distance from a starting point walking at various speeds and time.
   b. Using the motion detector as the starting point, walk away from the motion detector to make a graph on the graphing calculator that matches the one you drew.
   c. Compare your walking speeds to each change in steepness on the graph.

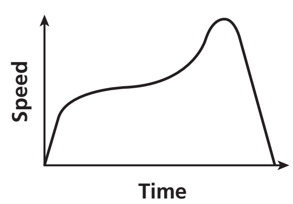
LINK

Sports



On November 1, 1938, the underdog Seabiscuit beat the heavily favored Triple-Crown winner War Admiral in a historic horse race at Pimlico Race Course in Baltimore, Maryland.

18. **Sports** The graph shows the speed of a horse during and after a race. Use it to describe the changing pace of the horse during the race.



19. **Recreation** You hike up a mountain path starting at 10 A.M. You camp overnight and then walk back down the same path at the same pace at 10 A.M. the next morning. On the same set of axes, graph the relationship between distance from the top of the mountain and the time of day for both the hike up and the hike down. What does the point of intersection of the graphs represent?

20. **Critical Thinking** Suppose that you sketched a graph of speed related to time for a brick that fell from the top of a building. Then you sketched a graph for speed related to time for a ball that was rolled down a hill and then came to rest. How would the graphs be the same? How would they be different?

H.O.T. 21. **Write About It** Describe a real-life situation that could be represented by a distinct graph. Then describe a real-life situation that could be represented by a continuous graph.

Real-World Connections

22. A rectangular pool that is 4 feet deep at all places is being filled at a constant rate.
   a. Sketch a graph to show the depth of the water as it increases over time.
   b. The side view of another swimming pool is shown. If the pool is being filled at a constant rate, sketch a graph to show the depth of the water as it increases over time.

TEST PREP

23. Which situation would NOT be represented by a discrete graph?

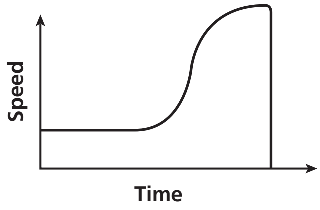
Ⓐ Amount of money earned based on the number of cereal bars sold

Ⓑ Number of visitors to a grocery store per day for one week

Ⓒ The amount of iced tea in a pitcher at a restaurant during the lunch hour

Ⓓ The total cost of buying 1, 2, or 3 CDs at the music store

24. Which situation is best represented by the graph?



Ⓕ A snowboarder starts at the bottom of the hill and takes a ski lift to the top.

Ⓖ A cruise boat travels at a steady pace from the port to its destination.

Ⓗ An object falls from the top of a building and gains speed at a rapid pace before hitting the ground.

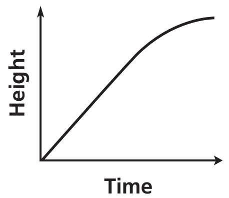
Ⓙ A marathon runner starts at a steady pace and then runs faster at the end of the race before stopping at the finish line.

H.O.T. **25.** **Short Response** Marla participates in a triathlon consisting of swimming, biking, and running. Would a graph of Marla's speed during the triathlon be a continuous graph or a distinct graph? Explain.

CHALLENGE AND EXTEND

Pictured are three vases and graphs representing the height of water as it is poured into each of the vases at a constant rate. Match each vase with the correct graph.

26.



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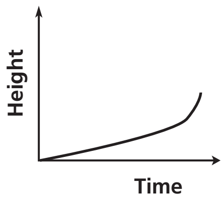

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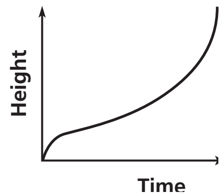
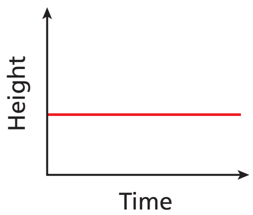


MATHEMATICAL PRACTICES

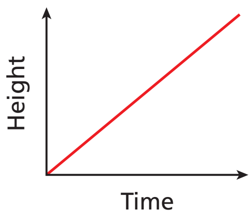
FOCUS ON MATHEMATICAL PRACTICES

H.O.T. **29.** **Modeling** As Kayla burns a candle, she records the amount of time that passes and the height of the burning candle. Which graph could reasonably represent her data? Explain your choice.

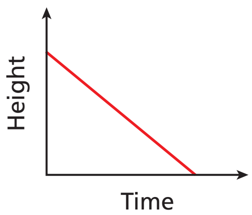
Graph A



Graph B



Graph C



8-2 Relations and Functions

Essential Question: How can you identify the domain and range of a relation and tell whether a relation is a function?

Objectives
Identify functions.
Find the domain and range of relations and functions.

Vocabulary
relation
domain
range
function

Why learn this?
You can use a relation to show finishing positions and scores in a track meet.

Previously, you saw relationships represented by graphs. Relationships can also be represented by a set of ordered pairs, called a **relation**.

In the scoring system of some track meets, first place is worth 5 points, second place is worth 3 points, third place is worth 2 points, and fourth place is worth 1 point. This scoring system is a relation, so it can be shown as ordered pairs, $\{(1, 5), (2, 3), (3, 2), (4, 1)\}$. You can also show relations in other ways, such as tables, graphs, or *mapping diagrams*.

COMMON CORE GPS **EXAMPLE 1** Prep. for MCC9-12.F.IF.1

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Showing Multiple Representations of Relations

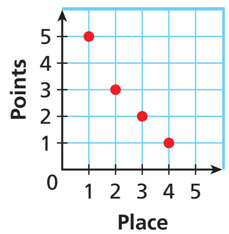
Express the relation for the track meet scoring system, $\{(1, 5), (2, 3), (3, 2), (4, 1)\}$, as a table, as a graph, and as a mapping diagram.

Table

Track Scoring	
Place	**Points**
1	5
2	3
3	2
4	1

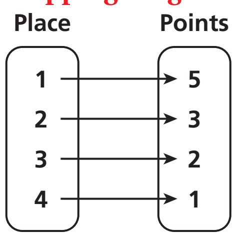
Write all x-values under "Place" and all y-values under "Points."

Graph



Use the x- and y-values to plot the ordered pairs.

Mapping Diagram



Write all x-values under "Place" and all y-values under "Points." Draw an arrow from each x-value to its corresponding y-value.

CHECK IT OUT!

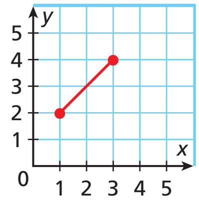
1. Express the relation $\{(1, 3)\ (2, 4), (3, 5)\}$ as a table, as a graph, and as a mapping diagram.

The **domain** of a relation is the set of first coordinates (or x-values) of the ordered pairs. The **range** of a relation is the set of second coordinates (or y-values) of the ordered pairs. The domain of the track meet scoring system is $\{1, 2, 3, 4\}$. The range is $\{5, 3, 2, 1\}$.

COMMON CORE GPS **EXAMPLE 2** MCC9-12.F.IF.5

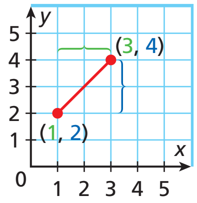
Finding the Domain and Range of a Relation

Give the domain and range of the relation.



The domain is all x-values from 1 through 3, inclusive.

The range is all y-values from 2 through 4, inclusive.



D: $1 \le x \le 3$ R: $2 \le y \le 4$

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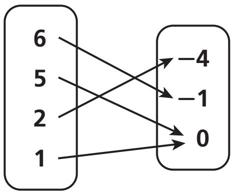
CHECK IT OUT! **Give the domain and range of each relation.**

2a.



2b.

x	y
1	1
4	4
8	1

A **function** is a special type of relation that pairs each domain value with exactly one range value.

COMMON CORE GPS **EXAMPLE 3** MCC.9-12.F.IF.1

Identifying Functions

Give the domain and range of each relation. Tell whether the relation is a function. Explain.

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Writing Math

When there is a finite number of values in a domain or range, list the values inside braces.

A

Field Trip

Students x	Buses y
75	2
68	2
125	3

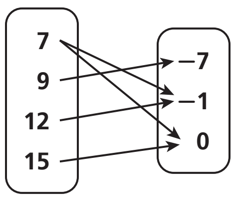
D: $\{75, 68, 125\}$

R: $\{2, 3\}$

Even though 2 appears twice in the table, it is written only once when writing the range.

This relation is a function. Each domain value is paired with exactly one range value.

B



Use the arrows to determine which domain values correspond to each range value.

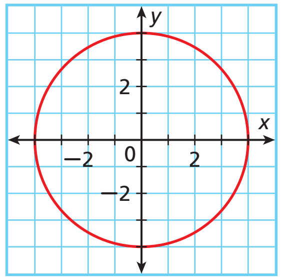
D: $\{7, 9, 12, 15\}$

R: $\{-7, -1, 0\}$

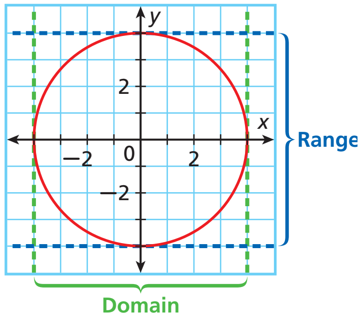
This relation is not a function. Each domain value does not have exactly one range value. The domain value 7 is paired with the range values -1 and 0.

Give the domain and range of each relation. Tell whether the relation is a function. Explain.

C



Draw lines to see the domain and range values.



D: $-4 \le x \le 4$ R: $-4 \le y \le 4$

x	4	0	0	−4
y	0	4	−4	0

To compare domain and range values, make a table using points from the graph.

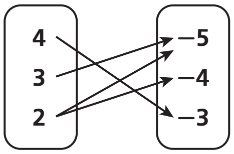
This relation is not a function because there are several domain values that have more than one range value. For example, the domain value 0 is paired with both 4 and −4.

Helpful Hint

To find the domain and range of a graph, it may help to draw lines to see the *x*- and *y*-values.

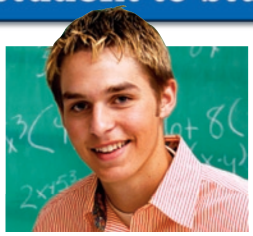
CHECK IT OUT! **Give the domain and range of each relation. Tell whether the relation is a function. Explain.**

3a. $\{(8, 2), (-4, 1), (-6, 2), (1, 9)\}$ **3b.**



Student to Student

Functions



Eric Dawson
Boone High School

*I decide whether a list of ordered pairs is a function by looking at the **x-values**. If they're all different, then it's a function.*

(1, 6), (2, 5), (6, 5), (0, 8)	*(5, 6), (7, 2), (5, 8), (6, 3)*
All different x-values	***Same x-value*** (with different *y*-values)
Function	*Not a function*

MCC.MP.3 MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Describe how to tell whether a set of ordered pairs is a function.

2. Can the graph of a vertical line segment represent a function? Explain.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer by explaining when a relation is a function and when it is not a function.

A relation is . . .	
A function if . . .	Not a function if . . .

8-2 Exercises

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Homework Help

GUIDED PRACTICE

Vocabulary **Apply the vocabulary from this lesson to answer each question.**

1. Use a mapping diagram to show a relation that is not a *function*.
2. The set of x-values for a relation is also called the __?__. (*domain* or *range*)

SEE EXAMPLE 1

Express each relation as a table, as a graph, and as a mapping diagram.

3. $\{(1, 1), (1, 2)\}$
4. $\left\{(-1, 1), \left(-2, \frac{1}{2}\right), \left(-3, \frac{1}{3}\right), \left(-4, \frac{1}{4}\right)\right\}$
5. $\{(-1, 1), (-3, 3), (5, -5), (-7, 7)\}$
6. $\{(0, 0), (2, -4), (2, -2)\}$

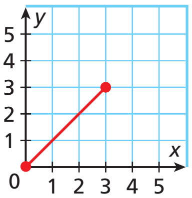
SEE EXAMPLE 2

Give the domain and range of each relation.

7. $\{(-5, 7), (0, 0), (2, -8), (5, -20)\}$
8. $\{(1, 2), (2, 4), (3, 6), (4, 8), (5, 10)\}$
9.

x	3	5	2	8	6
y	9	25	4	81	36

10.



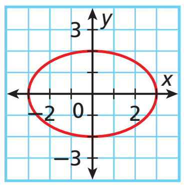
SEE EXAMPLE 3

Multi-Step **Give the domain and range of each relation. Tell whether the relation is a function. Explain.**

11. $\{(1, 3), (1, 0), (1, -2), (1, 8)\}$
12. $\{(-2, 1), (-1, 2), (0, 3), (1, 4)\}$
13.

x	−2	−1	0	1	2
y	1	1	1	1	1

14.



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
15–16	1
17–18	2
19–20	3

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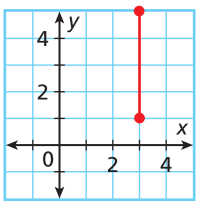
Online Extra Practice

Express each relation as a table, as a graph, and as a mapping diagram.

15. $\{(-2, -4), (-1, -1), (0, 0), (1, -1), (2, -4)\}$
16. $\left\{(2, 1), \left(2, \frac{1}{2}\right), (2, 2), \left(2, 2\frac{1}{2}\right)\right\}$

Give the domain and range of each relation.

17.

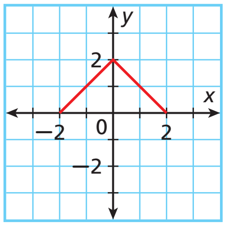


18.

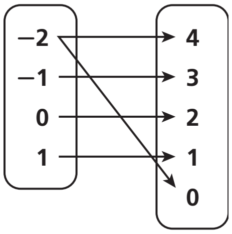
x	*y*
4	4
5	5
6	6
7	7
8	8

Multi-Step **Give the domain and range of each relation. Tell whether the relation is a function. Explain.**

19.



20.



21. **Consumer Application** An electrician charges a base fee of \$75 plus \$50 for each hour of work. Create a table that shows the amount the electrician charges for 1, 2, 3, and 4 hours of work. Let x represent the number of hours and y represent the amount charged for x hours. Is this relation a function? Explain.

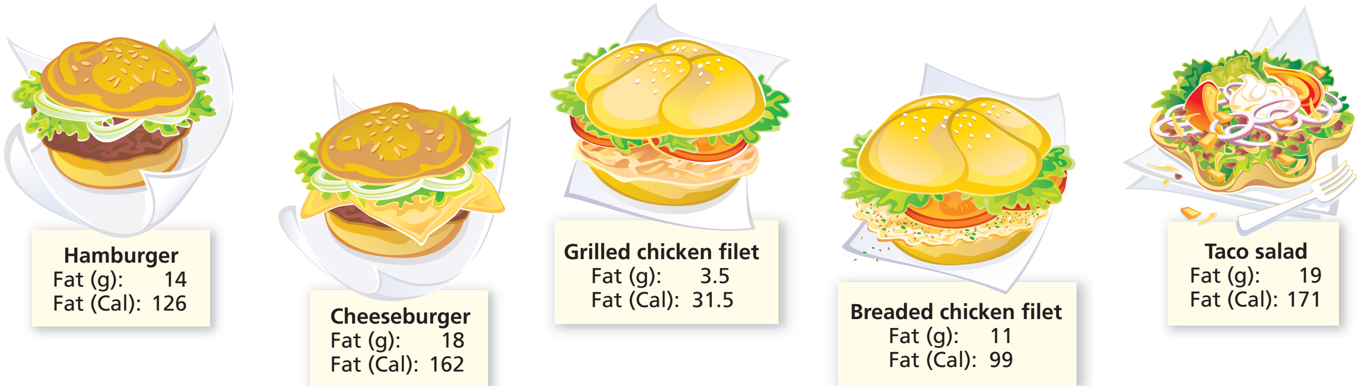
22. **Geometry** Write a relation as a set of ordered pairs in which the x-value represents the side length of a square and the y-value represents the area of that square. Use a domain of 2, 4, 6, 9, and 11.

23. **Multi-Step** Create a mapping diagram to display the numbers of days in 1, 2, 3, and 4 weeks. Is this relation a function? Explain.

24. **Nutrition** The illustrations list the number of grams of fat and the number of Calories from fat for selected foods.

a. Create a graph for the relation between grams of fat and Calories from fat.

b. Is this relation a function? Explain.



25. **Recreation** A shop rents canoes for a \$7 equipment fee plus \$2 per hour, with a maximum cost of \$15 per day. Express the number of hours x and the cost y as a relation in table form, and find the cost to rent a canoe for 1, 2, 3, 4, and 5 hours. Is this relation a function? Explain.

26. **Health** You can burn about 6 Calories per minute bicycling. Let x represent the number of minutes bicycled, and let y represent the number of Calories burned.

a. Write ordered pairs to show the number of Calories burned by bicycling for 60, 120, 180, 240, or 300 minutes. Graph the ordered pairs.

b. Find the domain and range of the relation.

c. Does this graph represent a function? Explain.

27. **Critical Thinking** For a function, can the number of elements in the range be greater than the number of elements in the domain? Explain.

28. **Critical Thinking** Tell whether each statement is true or false. If false, explain why.

a. All relations are functions.

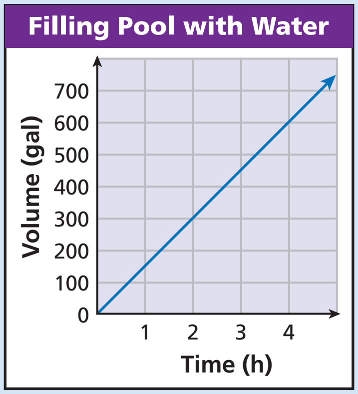
b. All functions are relations.

Real-World Connections

29. **a.** The graph shows the amount of water being pumped into a pool over a 5-hour time period. Find the domain and range.

b. Does the graph represent a function? Explain.

c. Give the time and volume as ordered pairs at 2 hours and at 3 hours 30 minutes.



H.O.T. **30.** **ERROR ANALYSIS** When asked whether the relation $\{(-4, 16), (-2, 4), (0, 0), (2, 4)\}$ is a function, a student stated that the relation is not a function because 4 appears twice. What error did the student make? How would you explain to the student why this relation is a function?

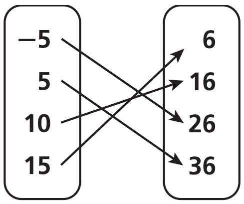
H.O.T. **31.** **Write About It** Describe a real-world situation for a relation that is NOT a function. Create a mapping diagram to show why the relation is not a function.

TEST PREP

32. Which of the following relations is NOT a function?

Ⓐ $\{(6, 2), (-1, 2), (-3, 2), (-5, 2)\}$

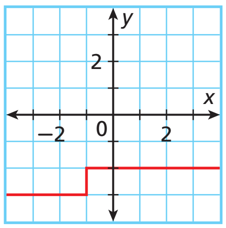
Ⓑ



Ⓒ

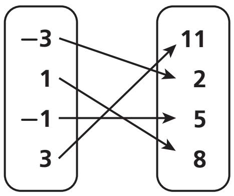
x	3	5	7
y	1	15	30

Ⓓ



33. Which is NOT a correct way to describe the function $\{(-3, 2), (1, 8), (-1, 5), (3, 11)\}$?

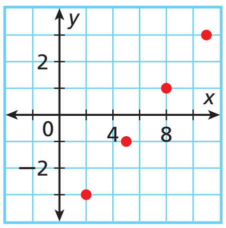
Ⓕ



Ⓗ Domain: $\{-3, 1, -1, 3\}$

Range: $\{2, 8, 5, 11\}$

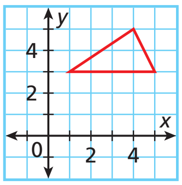
Ⓖ

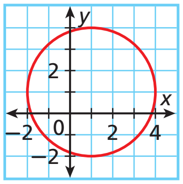


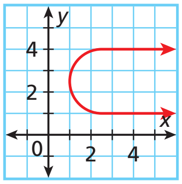
Ⓙ

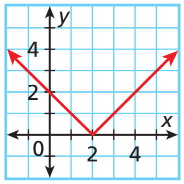
x	y
–3	2
–1	5
1	8
3	11

34. Which graph represents a function?

(A)


(B)


(C)


(D)


35. **Extended Response** Use the table for the following.

x	−3	−1	0	1	3
y	5	7	9	11	13

a. Express the relation as ordered pairs.

b. Give the domain and range of the relation.

c. Does the relation represent a function? Explain your answer.

CHALLENGE AND EXTEND

36. What values of a make the relation $\{(a, 1), (2, 3), (4, 5)\}$ a function? Explain.

37. What values of b make the relation $\{(5, 6), (7, 8), (9, b)\}$ a function? Explain.

38. The *inverse* of a relation is created by interchanging the x- and y- coordinates of each ordered pair in the relation.

a. Find the inverse of the following relation: $\{(-2, 5), (0, 4), (3, -8), (7, 5)\}$.

b. Is the original relation a function? Why or why not? Is the inverse of the relation a function? Why or why not?

c. The statement "If a relation is a function, then the inverse of the relation is also a function" is sometimes true. Give an example of a relation and its inverse that are both functions. Then give an example of a relation and its inverse that are both not functions.

MATHEMATICAL PRACTICES

FOCUS ON MATHEMATICAL PRACTICES

H.O.T. **39.** **Analysis** Gina surveys 15 high school students, asking them what grade they're in and how much they spent that day on lunch. She makes a table of the data, and then draws a graph. She labels her x-axis *Grade Level* and her y-axis *Cost of Lunch*.

a. What is the domain of her graph?

b. Can you find the range of the graph from the information given? If not, describe the range in general terms.

c. What is the maximum possible number of elements in the range? What is the minimum?

d. Is it likely that the graph is a function? Explain your answer.

H.O.T. **40.** **Error Analysis** Nick matches each day of the week to anyone in his class of 20 students that was born on that day. He says this is a function. Oscar says this relation is not a function.

a. Who do you think is correct? Explain why.

b. How could Nick relate the same two sets in a way that creates a function?

8-2 Algebra TASK

Use with Relations and Functions

The Vertical-Line Test

The *vertical-line test* can be used to visually determine whether a graphed relation is a function.

MATHEMATICAL PRACTICES **Look for and express regularity in repeated reasoning.**

MCC9-12.F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. . . .

Activity

Table 1

x	y
−2	−5
−1	−3
0	−1
1	1
2	3
3	5

1. Look at the values in Table 1. Is every x-value paired with exactly one y-value? If not, what x-value(s) are paired with more than one y-value?
2. Is the relation a function? Explain.
3. Graph the points from the Table 1. Draw a vertical line through each point of the graph. Does any vertical line touch more than one point?

Table 2

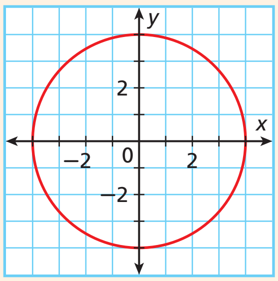
x	y
−2	−3
1	4
0	5
1	2
2	3
3	5

4. Look at the values in Table 2. Is every x-value paired with exactly one y-value? If not, what x-value(s) are paired with more than one y-value?
5. Is the relation a function? Explain.
6. Graph the points from the Table 2. Draw a vertical line through each point of the graph. Does any vertical line touch more than one point?
7. What is the x-value of the two points that are on the same vertical line? Is that x-value paired with more than one y-value?
8. Write a statement describing how to use a vertical line to tell if a relation is a function. This is called the vertical-line test.
9. Why does the vertical-line test work?

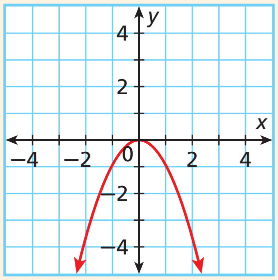
Try This

Use the vertical-line test to determine whether each relation is a function. If a relation is not a function, list two ordered pairs that show the same x-value with two different y-values.

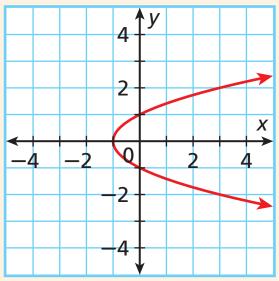
1.



2.



3.



8-3 Algebra TASK

Use with Writing Functions

Model Variable Relationships

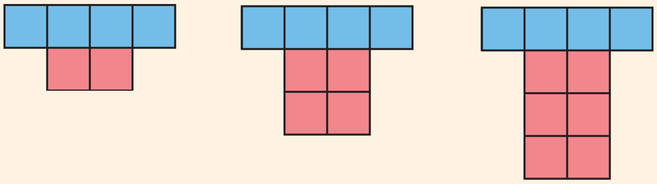
You can use models to represent an algebraic relationship. Using these models, you can write an algebraic expression to help describe and extend patterns.

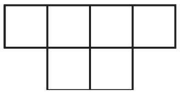
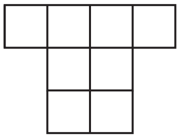
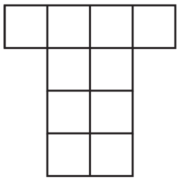
MATHEMATICAL PRACTICES **Look for and express regularity in repeated reasoning.**

The diagrams below represent the side views of tables. Each has a tabletop and a base. Copy and complete the chart using the pattern shown in the diagrams.

Tabletop →

Base →



TERM NUMBER	FIGURE	DESCRIPTION OF FIGURE	EXPRESSION FOR NUMBER OF BLOCKS	VALUE OF TERM (NUMBER OF BLOCKS)	ORDERED PAIR
1		length of tabletop = 4 height of base = 1	$4 + (2)1$	6	$(1, 6)$
2		length of tabletop = 4 height of base = 2		8	
3		length of tabletop = 4 height of base = 3		10	
4					
5					
n					

Try This

1. Explain why you must multiply the height of the base by 2.
2. What does the ordered pair (1, 6) mean?
3. Does the ordered pair (10, 24) belong in this pattern? Why or why not?
4. Which expression from the table describes how you would find the total number of blocks for any term number n?
5. Use your rule to find the 25th term in this pattern.

8-3 Writing Functions

Essential Question: How can you use function notation to write and evaluate functions?

Objectives

Identify independent and dependent variables.

Write an equation in function notation and evaluate a function for given input values.

Vocabulary

independent variable
dependent variable
function rule
function notation

Why learn this?

You can use a function rule to calculate how much money you will earn for working specific amounts of time.

Suppose Tasha baby-sits and charges $5 per hour.

Time Worked (h) x	1	2	3	4
Amount Earned ($) y	5	10	15	20

The amount of money Tasha earns is $5 times the number of hours she works. Write an equation using two different variables to show this relationship.

Amount earned is $5 times the number of hours worked.

$$y = 5 \cdot x$$

Tasha can use this equation to find how much money she will earn for any number of hours she works.

COMMON CORE GPS **EXAMPLE 1** MCC9-12.A.CED.2

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Using a Table to Write an Equation

Determine a relationship between the x- and y-values. Write an equation.

x	1	2	3	4
y	−2	−1	0	1

Step 1 List possible relationships between the first x- and y-values.

$1 - 3 = -2$ or $1(-2) = -2$

Step 2 Determine if one relationship works for the remaining values.

$2 - 3 = -1$ ✓ $\quad 2(-2) \neq -1$ ✗

$3 - 3 = 0$ ✓ $\quad 3(-2) \neq 0$ ✗

$4 - 3 = 1$ ✓ $\quad 4(-2) \neq 1$ ✗

The first relationship works. The value of y is 3 less than x.

Step 3 Write an equation.

$y = x - 3$ *The value of y is 3 less than x.*

CHECK IT OUT!

1. Determine a relationship between the x- and y-values in the relation $\{(1, 3), (2, 6), (3, 9), (4, 12)\}$. Write an equation.

The equation in Example 1 describes a function because for each x-value (input), there is only one y-value (output).

RubberBall/Alamy

The **input** of a function is the **independent variable**. The **output** of a function is the **dependent variable**. The value of the dependent variable *depends* on, or is a function of, the value of the independent variable. For Tasha, the amount she earns depends on, or is a function of, the amount of time she works.

COMMON CORE GPS **EXAMPLE 2** MCC9-12.F.IF.1

Identifying Independent and Dependent Variables

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Identify the independent and dependent variables in each situation.

A **In the winter, more electricity is used when the temperature goes down, and less is used when the temperature rises.**

The **amount of electricity** used *depends on* the **temperature**.

Dependent: **amount of electricity** Independent: **temperature**

B **The cost of shipping a package is based on its weight.**

The **cost** of shipping a package *depends on* its **weight**.

Dependent: **cost** Independent: **weight**

C **The faster Ron walks, the quicker he gets home.**

The **time** it takes Ron to get home *depends on* the **speed** he walks.

Dependent: **time** Independent: **speed**

Helpful Hint

There are several different ways to describe the variables of a function.

Independent Variable	Dependent Variable
x-values	y-values
Domain	Range
Input	Output
x	$f(x)$

CHECK IT OUT! **Identify the independent and dependent variables in each situation.**

2a. A company charges \$10 per hour to rent a jackhammer.

2b. Apples cost \$0.99 per pound.

An algebraic expression that defines a function is a **function rule**. $5 \cdot x$ in the equation about Tasha's earnings is a function rule.

If x is the independent variable and y is the dependent variable, then **function notation** for y is $f(x)$, read "f of x," where f names the function. When an equation in two variables describes a function, you can use function notation to write it.

The dependent variable	**is**	**a function of**	**the independent variable**.
y	**is**	**a function of**	x.
y	$=$	f	(x)

Since $y = f(x)$, Tasha's earnings, $y = 5x$, can be rewritten in function notation by substituting $f(x)$ for y: $f(x) = 5x$. Sometimes functions are written using y, and sometimes functions are written using $f(x)$.

COMMON CORE GPS **EXAMPLE 3** MCC9-12.F.IF.2

Writing Functions

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Identify the independent and dependent variables. Write an equation in function notation for each situation.

A **A lawyer's fee is \$200 per hour for her services.**

The **fee** for the lawyer depends on how many **hours** she works.

Dependent: **fee** Independent: **hours**

Let h represent the number of hours the lawyer works.

The function for the lawyer's fee is $f(h) = 200h$.

Identify the independent and dependent variables. Write an equation in function notation for each situation.

B **The admission fee to a local carnival is \$8. Each ride costs \$1.50.**

The total cost depends on the number of rides ridden, plus \$8.

Dependent: total cost   Independent: number of rides

Let r represent the number of rides ridden.

The function for the total cost of the carnival is $f(r) = 1.50r + 8$.

CHECK IT OUT! **Identify the independent and dependent variables. Write an equation in function notation for each situation.**

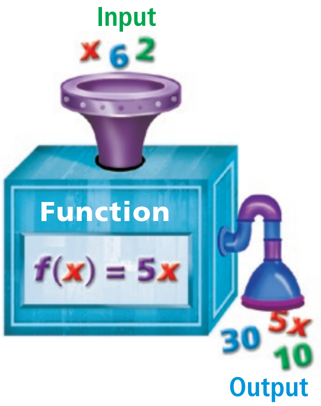
3a. Steven buys lettuce that costs \$1.69/lb.

3b. An amusement park charges a \$6.00 parking fee plus \$29.99 per person.

Animated Math

You can think of a function as an input-output machine. For Tasha's earnings, $f(x) = 5x$, if you input a value x, the output is $5x$.

If Tasha wanted to know how much money she would earn by working 6 hours, she could input 6 for x and find the output. This is called *evaluating the function.*



COMMON CORE GPS **EXAMPLE 4** MCC9-12.F.IF.2

Evaluating Functions

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Evaluate each function for the given input values.

A **For $f(x) = 5x$, find $f(x)$ when $x = 6$ and when $x = 7.5$.**

$$\begin{aligned} f(x) &= 5x \\ f(6) &= 5(6) && \textit{Substitute 6 for x.} \\ &= 30 && \textit{Simplify.} \end{aligned}$$

$$\begin{aligned} f(x) &= 5x \\ f(7.5) &= 5(7.5) && \textit{Substitute 7.5 for x.} \\ &= 37.5 && \textit{Simplify.} \end{aligned}$$

B **For $g(t) = 2.30t + 10$, find $g(t)$ when $t = 2$ and when $t = -5$.**

$$\begin{aligned} g(t) &= 2.30t + 10 \\ g(2) &= 2.30(2) + 10 \\ &= 4.6 + 10 \\ &= 14.6 \end{aligned}$$

$$\begin{aligned} g(t) &= 2.30t + 10 \\ g(-5) &= 2.30(-5) + 10 \\ &= -11.5 + 10 \\ &= -1.5 \end{aligned}$$

C **For $h(x) = \frac{1}{2}x - 3$, find $h(x)$ when $x = 12$ and when $x = -8$.**

$$\begin{aligned} h(x) &= \frac{1}{2}x - 3 \\ h(12) &= \frac{1}{2}(12) - 3 \\ &= 6 - 3 \\ &= 3 \end{aligned}$$

$$\begin{aligned} h(x) &= \frac{1}{2}x - 3 \\ h(-8) &= \frac{1}{2}(-8) - 3 \\ &= -4 - 3 \\ &= -7 \end{aligned}$$

Reading Math

Functions can be named with any letter; *f*, *g*, and *h* are the most common. You read $f(6)$ as "*f* of 6," and $g(2)$ as "*g* of 2."

CHECK IT OUT! **Evaluate each function for the given input values.**

4a. For $h(c) = 2c - 1$, find $h(c)$ when $c = 1$ and $c = -3$.

4b. For $g(t) = \frac{1}{4}t + 1$, find $g(t)$ when $t = -24$ and $t = 400$.

When a function describes a real-world situation, every real number is not always reasonable for the domain and range. For example, a number representing the length of an object cannot be negative, and only whole numbers can represent a number of people.

COMMON CORE GPS **EXAMPLE 5** MCC9-12.F.IF.5

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Finding the Reasonable Domain and Range of a Function

Manuel has already sold \$20 worth of tickets to the school play. He has 4 tickets left to sell at \$2.50 per ticket. Write a function to describe how much money Manuel can collect from selling tickets. Find the reasonable domain and range for the function.

Money collected from ticket sales	is	\$2.50	per	ticket	plus	the \$20 already sold.
$f(x)$	$=$	\$2.50	$\cdot$	x	$+$	20

If he sells x more tickets, he will have collected $f(x) = 2.50x + 20$ dollars.

Manuel has only 4 tickets left to sell, so he could sell 0, 1, 2, 3, or 4 tickets. A reasonable domain is $\{0, 1, 2, 3, 4\}$.

Substitute these values into the function rule to find the range values.

x	0	1	2	3	4
$f(x)$	$2.50(0) + 20 = 20$	$2.50(1) + 20 = 22.50$	$2.50(2) + 20 = 25$	$2.50(3) + 20 = 27.50$	$2.50(4) + 20 = 30$

The reasonable range for this situation is $\{\$20, \$22.50, \$25, \$27.50, \$30\}$.

CHECK IT OUT!

5. The settings on a space heater are the whole numbers from 0 to 3. The total number of watts used for each setting is 500 times the setting number. Write a function to describe the number of watts used for each setting. Find the reasonable domain and range for the function.

MCC.MP.4, MCC.MP.6 MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. When you input water into an ice machine, the output is ice cubes. Name another real-world object that has an input and an output.

2. How do you identify the independent and dependent variables in a situation?

3. Explain how to find reasonable domain values for a function.

Know it! Note

4. GET ORGANIZED Copy and complete the graphic organizer. Use the function $y = x + 3$ and the domain $\{-2, -1, 0, 1, 2\}$.

Ways to Represent Functions

Equation in function notation | Table | Graph

8-3 Exercises

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Homework Help

GUIDED PRACTICE

Vocabulary **Apply the vocabulary from this lesson to answer each question.**

1. The output of a function is the __?__ variable. (*independent* or *dependent*)
2. An algebraic expression that defines a function is a __?__. (*function rule* or *function notation*)

SEE EXAMPLE 1

Determine a relationship between the x- and y-values. Write an equation.

3.

x	1	2	3	4
y	−1	0	1	2

4. $\{(1, 4), (2, 7), (3, 10), (4, 13)\}$

SEE EXAMPLE 2

Identify the independent and dependent variables in each situation.

5. A small-size bottle of water costs \$1.99 and a large-size bottle of water costs \$3.49.
6. An employee receives 2 vacation days for every month worked.

SEE EXAMPLE 3

Identify the independent and dependent variables. Write an equation in function notation for each situation.

7. An air-conditioning technician charges customers \$75 per hour.
8. An ice rink charges \$3.50 for skates and \$1.25 per hour.

SEE EXAMPLE 4

Evaluate each function for the given input values.

9. For $f(x) = 7x + 2$, find $f(x)$ when $x = 0$ and when $x = 1$.
10. For $g(x) = 4x - 9$, find $g(x)$ when $x = 3$ and when $x = 5$.
11. For $h(t) = \frac{1}{3}t - 10$, find $h(t)$ when $t = 27$ and when $t = -15$.

SEE EXAMPLE 5

12. A construction company uses beams that are 2, 3, or 4 meters long. The measure of each beam must be converted to centimeters. Write a function to describe the situation. Find the reasonable domain and range for the function. (*Hint*: 1 m = 100 cm)

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
13–14	1
15–16	2
17–19	3
20–22	4
23	5

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Online Extra Practice

Determine a relationship between the x- and y-values. Write an equation.

13.

x	1	2	3	4
y	−2	−4	−6	−8

14. $\{(1, -1), (2, -2), (3, -3), (4, -4)\}$

Identify the independent and dependent variables in each situation.

15. Gardeners buy fertilizer according to the size of a lawn.
16. The cost to gift wrap an order is \$3 plus \$1 per item wrapped.

Identify the independent and dependent variables. Write an equation in function notation for each situation.

17. To rent a DVD, a customer must pay \$3.99 plus \$0.99 for every day that it is late.
18. Stephen charges \$25 for each lawn he mows.
19. A car can travel 28 miles per gallon of gas.

Evaluate each function for the given input values.

20. For $f(x) = x^2 - 5$, find $f(x)$ when $x = 0$ and when $x = 3$.

21. For $g(x) = x^2 + 6$, find $g(x)$ when $x = 1$ and when $x = 2$.

22. For $f(x) = \frac{2}{3}x + 3$, find $f(x)$ when $x = 9$ and when $x = -3$.

23. A mail-order company charges \$5 per order plus \$2 per item in the order, up to a maximum of 4 items. Write a function to describe the situation. Find the reasonable domain and range for the function.

LINK

Transportation

Air Force One refers to two specially configured Boeing 747-200B airplanes. The radio call sign when the president is aboard either aircraft or any Air Force aircraft is "Air Force One."

24. Transportation Air Force One can travel 630 miles per hour. Let h be the number of hours traveled. The function $d = 630h$ gives the distance d in miles that Air Force One travels in h hours.

a. Identify the independent and dependent variables. Write $d = 630h$ using function notation.

b. What are reasonable values for the domain and range in the situation described?

c. How far can Air Force One travel in 12 hours?

25. Complete the table for $g(z) = 2z - 5$.

z	1	2	3	4
$g(z)$				

26. Complete the table for $h(x) = x^2 + x$.

x	0	1	2	3
$h(x)$				

27. Estimation For $f(x) = 3x + 5$, estimate the output when $x = -6.89$, $x = 1.01$, and $x = 4.67$.

28. Transportation A car can travel 30 miles on a gallon of gas and has a 20-gallon gas tank. Let g be the number of gallons of gas the car has in its tank. The function $d = 30g$ gives the distance d in miles that the car travels on g gallons.

a. What are reasonable values for the domain and range in the situation described?

b. How far can the car travel on 12 gallons of gas?

H.O.T. **29. Critical Thinking** Give an example of a real-life situation for which the reasonable domain consists of 1, 2, 3, and 4 and the reasonable range consists of 2, 4, 6, and 8.

H.O.T. **30. ERROR ANALYSIS** Rashid saves \$150 each month. He wants to know how much he will have saved in 2 years. He writes the rule $s = m + 150$ to help him figure out how much he will save, where s is the amount saved and m is the number of months he saves. Explain why his rule is incorrect.

31. Write About It Give a real-life situation that can be described by a function. Identify the independent variable and the dependent variable.

Real-World Connections

32. The table shows the volume v of water pumped into a pool after t hours.

a. Determine a relationship between the time and the volume of water and write an equation.

b. Identify the independent and dependent variables.

c. If the pool holds 10,000 gallons, how long will it take to fill?

Amount of Water in Pool

Time (h)	Volume (gal)
0	0
1	1250
2	2500
3	3750
4	5000

TEST PREP

33. Marsha buys x pens at \$0.70 per pen and one pencil for \$0.10. Which function gives the total amount Marsha spends?

Ⓐ $c(x) = 0.70x + 0.10x$
Ⓑ $c(x) = 0.70x + 1$
Ⓒ $c(x) = (0.70 + 0.10)x$
Ⓓ $c(x) = 0.70x + 0.10$

34. Belle is buying pizzas for her daughter's birthday party, using the prices in the table. Which equation best describes the relationship between the total cost c and the number of pizzas p?

Pizzas	Total Cost (\$)
5	26.25
10	52.50
15	78.75

Ⓕ $c = 26.25p$
Ⓖ $c = 5.25p$
Ⓗ $c = p + 26.25$
Ⓙ $c = 6p - 3.75$

35. **Gridded Response** What is the value of $f(x) = 5 - \frac{1}{2}x$ when $x = 3$?

CHALLENGE AND EXTEND

36. The formula to convert a temperature that is in degrees Celsius x to degrees Fahrenheit $f(x)$ is $f(x) = \frac{9}{5}x + 32$. What are reasonable values for the domain and range when you convert to Fahrenheit the temperature of water as it rises from 0° to 100° Celsius?

37. **Math History** In his studies of the motion of free-falling objects, Galileo Galilei found that regardless of its mass, an object will fall a distance d that is related to the square of its travel time t in seconds. The modern formula that describes free-fall motion is $d = \frac{1}{2}gt^2$, where g is the acceleration due to gravity and t is the length of time in seconds the object falls. Find the distance an object falls in 3 seconds. (*Hint*: Research to find acceleration due to gravity in meters per second squared.)

MATHEMATICAL PRACTICES

FOCUS ON MATHEMATICAL PRACTICES

H.O.T. **38.** **Problem Solving** Alejandro's grandmother gives him \$25 dollars to start his savings account. For every dollar d he adds to this account, his grandmother puts in an additional \$5.

a. Write an equation that describes t, the total in his savings account, after he has added d dollars.

b. What is the independent variable for this equation? What is the dependent variable? Explain how you know.

c. Write the equation from part a in function notation.

H.O.T. **39.** **Modeling** Jasmine's monthly cell phone bill varies according to the number of minutes she uses. The table shows the relationship.

Minutes	0	100	200	300	400
Monthly Cost	\$15	\$25	\$35	\$45	\$55

Write a function in the form $f(x) = mx + b$ that relates the number of minutes Jasmine uses to her monthly cost. What does the value of m mean in the context of Jasmine's bill? What about the value of b?

H.O.T. **40.** **Analysis** You can perform basic operations, such as addition, subtraction, multiplication, and division, with functions. For example, when $f(x) = 4x$ and $g(x) = 2x - 1$, $f(x) + g(x) = 4x + (2x - 1) = 6x - 1$. Find $f(x) - g(x)$.

Ready to Go On?

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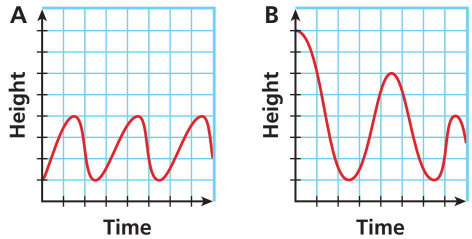
Assessment and Intervention

8-1 Graphing Relationships

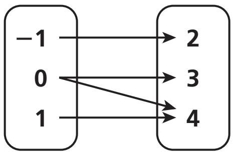


Choose the graph that best represents each situation.

1. A person bungee jumps from a high platform.
2. A person jumps on a trampoline in a steady motion.
3. Xander takes a quiz worth 100 points. Each question is worth 20 points. Sketch a graph to show his score if he misses 1, 2, 3, 4, or 5 questions.

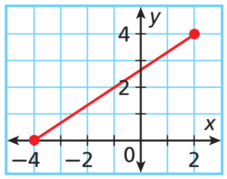
8-2 Relations and Functions

Give the domain and range of each relation. Tell whether the relation is a function. Explain.

4. 

5.

x	−2	−2	0	2	2
y	3	3	3	3	3

6. 

7. A local parking garage charges \$5.00 for the first hour plus \$1.50 for each additional hour or part of an hour. Write a relation as a set of ordered pairs in which the x-value represents the number of hours and the y-value represents the cost for x hours. Use a domain of 1, 2, 3, 4, 5. Is this relation a function? Explain.
8. A baseball coach is taking the team for ice cream. Four students can ride in each car. Create a mapping diagram to show the number of cars needed to transport 8, 10, 14, and 16 students. Is this relation a function? Explain.

8-3 Writing Functions

Determine a relationship between the x- and y-values. Write an equation.

9.

x	1	2	3	4
y	−6	−5	−4	−3

10.

x	1	2	3	4
y	−3	−6	−9	−12

11. A printer can print 8 pages per minute. Identify the dependent and independent variables for the situation. Write an equation in function notation.

Evaluate each function for the given input values.

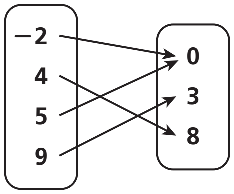
12. For $f(x) = 3x - 1$, find $f(x)$ when $x = 2$.
13. For $g(x) = x^2 - x$, find $g(x)$ when $x = -2$.
14. A photographer charges a sitting fee of \$15 plus \$3 for each pose. Write a function to describe the situation. Find a reasonable domain and range for up to 5 poses.

PARCC Assessment Readiness

COMMON CORE GPS

Selected Response

1. Give the domain and range of the relation.



Ⓐ D: {−2, 4, 5, 9}; R: {0, 3, 8}

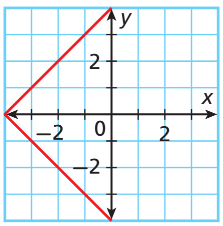
Ⓑ D: {0, 3, 8}; R: {−2, 4, 5, 9}

Ⓒ D: $-2 < x < 9$; R: $0 < x < 8$

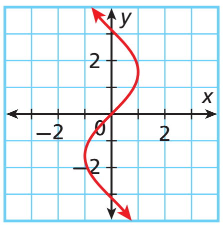
Ⓓ D: $-2 \leq x \leq 9$; R: $0 \leq x \leq 8$

2. Which graph represents a function?

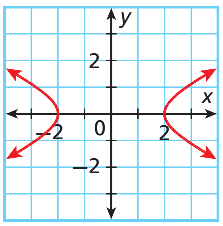
Ⓕ



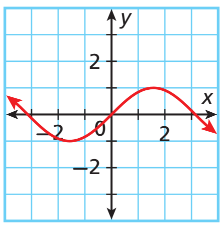
Ⓖ



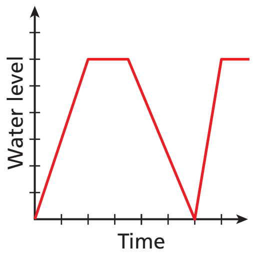
Ⓗ



Ⓙ



3. Write a possible situation for the graph.



Ⓐ A pool is filled with water, and people are having fun swimming and jumping in and out of the pool.

Ⓑ A pool is filled with water using one valve. Shortly after it is full, the pool needs to be emptied. Then it is refilled immediately, using two valves this time.

Ⓒ A pool is filled with water using one valve. Immediately after it is full, the pool needs to be emptied. It is refilled immediately after it is completely empty, using two valves this time.

Ⓓ A pool is filled with water. Shortly after it is full, the pool needs to be emptied. It is refilled immediately after it is completely empty, using one valve.

4. Determine a relationship between the *x*- and *y*-values. Write an equation.

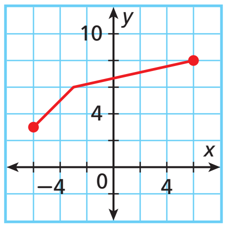
x	1	2	3	4
y	4	5	6	7

Ⓕ $y = -x + 3$ Ⓗ $y = x + 3$

Ⓖ $y = x + 4$ Ⓙ $y = 3x + 1$

Mini-Task

5. A function is graphed below.



What are the domain and range of the function?