

Special Systems and Systems of Inequalities



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The Common Core Georgia Performance Standards for Mathematical Practice describe varieties of expertise that all students should seek to develop.

Opportunities to develop these practices are integrated throughout this program.

- 1 Make sense of problems and persevere in solving them.
- 2 Reason abstractly and quantitatively.
- 3 Construct viable arguments and critique the reasoning of others.
- 4 Model with mathematics.
- 5 Use appropriate tools strategically.
- 6 Attend to precision.
- 7 Look for and make use of structure.
- 8 Look for and express regularity in repeated reasoning.

Unpacking the Standards



Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this chapter.



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Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Key Vocabulary

half-plane (*semiplano*) The part of the coordinate plane on one side of a line, which may include the line.

solution of a linear inequality in two variables (*solución de una desigualdad lineal en dos variables*)

An ordered pair or ordered pairs that make the inequality true.

system of linear inequalities (*sistema de desigualdades lineales*) A system of inequalities in which all of the inequalities are linear.



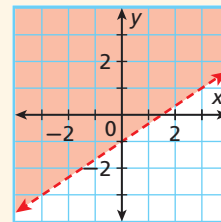
What It Means For You

Systems of linear inequalities model many real-life situations where you want to know when one or more conditions are met, but where there are many possible solutions.

EXAMPLE

Linear Inequality

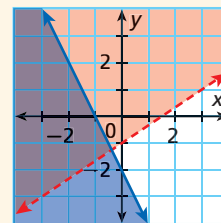
$$y > \frac{2}{3}x - 1$$



EXAMPLE

System of Two Linear Inequalities

$$\begin{cases} y > \frac{2}{3}x - 1 \\ y \leq -2x - 2 \end{cases}$$



EXAMPLE

System of Three Linear Inequalities

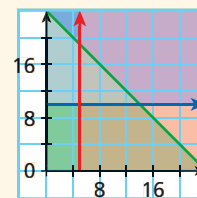
Tracy works at least 5 hours per week as a cashier: $c \geq 5$

Tracy works at least 10 hours per week at a library: $p \geq 10$

Tracy works at most 24 hours per week: $c + p \leq 24$

How can Tracy divide her time between the two jobs?

| Sample Solutions | |
|------------------|--------------|
| Cashier (hours) | Page (hours) |
| 5 | 10 |
| 5 | 16 |
| 8 | 12 |
| 8 | 16 |
| 10 | 12 |
| 12 | 12 |



7-1

Solving Special Systems

Essential Question: How can you solve consistent and inconsistent systems of linear equations?

Objectives

Solve special systems of linear equations in two variables.

Classify systems of linear equations and determine the number of solutions.

Vocabulary

consistent system
inconsistent system
independent system
dependent system

Why learn this?

Linear systems can be used to analyze business growth, such as comic book sales. (See Example 4.)

When two lines intersect at a point, there is exactly one solution to the system. A system with at least one solution is a **consistent system**.

When the two lines in a system do not intersect, they are parallel lines. There are no ordered pairs that satisfy both equations, so there is no solution. A system that has no solution is an **inconsistent system**.



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EXAMPLE
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1 Systems with No Solution

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Show that $\begin{cases} y = x - 1 \\ -x + y = 2 \end{cases}$ has no solution.

Method 1 Compare slopes and y-intercepts.

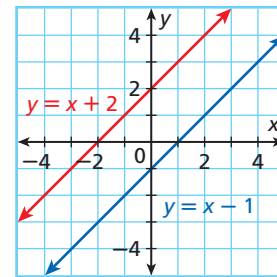
$$\begin{aligned} y = x - 1 &\rightarrow y = 1x - 1 \\ -x + y = 2 &\rightarrow y = 1x + 2 \end{aligned}$$

Write both equations in slope-intercept form. The lines are parallel because they have the same slope and different y-intercepts.

This system has no solution.

Method 2 Graph the system.

The lines are parallel.



This system has no solution.

Method 3 Solve the system algebraically. Use the substitution method.

$$\begin{aligned} -x + (x - 1) &= 2 && \text{Substitute } x - 1 \text{ for } y \text{ in the second equation, and solve.} \\ -1 &= 2 && \text{False} \end{aligned}$$

This system has no solution.



1. Show that $\begin{cases} y = -2x + 5 \\ 2x + y = 1 \end{cases}$ has no solution.

If two linear equations in a system have the same graph, the graphs are coincident lines, or the same line. There are infinitely many solutions of the system because every point on the line represents a solution of both equations.

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EXAMPLE
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2 Systems with Infinitely Many Solutions

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Show that $\begin{cases} y = 2x + 1 \\ 2x - y + 1 = 0 \end{cases}$ has infinitely many solutions.

Method 1 Compare slopes and y-intercepts.

$$\begin{aligned} y = 2x + 1 &\rightarrow y = 2x + 1 && \text{Write both equations in slope-intercept form. The lines have the same slope and the same y-intercept.} \\ 2x - y + 1 = 0 &\rightarrow y = 2x + 1 \end{aligned}$$

If this system were graphed, the graphs would be the same line. There are infinitely many solutions.

Method 2 Solve the system algebraically. Use the elimination method.

$$\begin{aligned} y = 2x + 1 &\rightarrow -2x + y = 1 && \text{Write equations to line up like terms.} \\ 2x - y + 1 = 0 &\rightarrow +2x - y = -1 && \text{Add the equations.} \\ \hline 0 = 0 &&& \text{True. The equation is an identity.} \end{aligned}$$

There are infinitely many solutions.

Caution!

$0 = 0$ is a true statement. It does not mean the system has zero solutions or no solution.



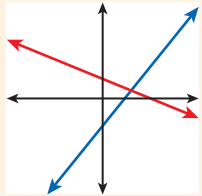
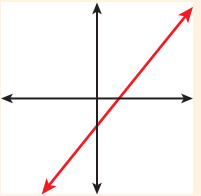
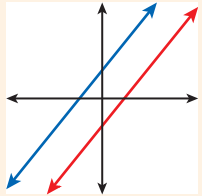
2. Show that $\begin{cases} y = x - 3 \\ x - y - 3 = 0 \end{cases}$ has infinitely many solutions.

Consistent systems can either be independent or dependent.

- An **independent system** has exactly one solution. The graph of an independent system consists of two intersecting lines.
- A **dependent system** has infinitely many solutions. The graph of a dependent system consists of two coincident lines.



Classification of Systems of Linear Equations

| CLASSIFICATION | CONSISTENT AND INDEPENDENT | CONSISTENT AND DEPENDENT | INCONSISTENT |
|----------------------------|--|---|---|
| Number of Solutions | Exactly one | Infinitely many | None |
| Description | Different slopes | Same slope, same y-intercept | Same slope, different y-intercepts |
| Graph | Intersecting lines  | Coincident lines  | Parallel lines  |

Classifying Systems of Linear Equations

Classify each system. Give the number of solutions.

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$$\text{A } \begin{cases} 2y = x + 2 \\ -\frac{1}{2}x + y = 1 \end{cases}$$

$$2y = x + 2 \rightarrow y = \frac{1}{2}x + 1$$

Write both equations in slope-intercept form.

$$-\frac{1}{2}x + y = 1 \rightarrow y = \frac{1}{2}x + 1$$

The lines have the same slope and the same y-intercepts. They are the same.

The system is consistent and dependent. It has infinitely many solutions.

$$\text{B } \begin{cases} y = 2(x - 1) \\ y = x + 1 \end{cases}$$

$$y = 2(x - 1) \rightarrow y = 2x - 2$$

Write both equations in slope-intercept form.

$$y = x + 1 \rightarrow y = 1x + 1$$

The lines have different slopes. They intersect.

The system is consistent and independent. It has one solution.



Classify each system. Give the number of solutions.

$$\text{3a. } \begin{cases} x + 2y = -4 \\ -2(y + 2) = x \end{cases}$$

$$\text{3b. } \begin{cases} y = -2(x - 1) \\ y = -x + 3 \end{cases}$$

$$\text{3c. } \begin{cases} 2x - 3y = 6 \\ y = \frac{2}{3}x \end{cases}$$

Business Application

The sales manager at Comics Now is comparing its sales with the sales of its competitor, Dynamo Comics. If the sales patterns continue, will the sales for Comics Now ever equal the sales for Dynamo Comics? Explain.

Comic Books Sold per Year (thousands)

| | 2005 | 2006 | 2007 | 2008 |
|---------------|------|------|------|------|
| Comics Now | 130 | 170 | 210 | 250 |
| Dynamo Comics | 180 | 220 | 260 | 300 |



Use the table to write a system of linear equations. Let y represent the sales total and x represent the number of years since 2005.

| | Sales total | equals | increase in sales per year | times | years | plus | beginning sales. |
|---------------|-------------|--------|----------------------------|---------|-------|------|------------------|
| Comics Now | y | $=$ | 40 | \cdot | x | $+$ | 130 |
| Dynamo Comics | y | $=$ | 40 | \cdot | x | $+$ | 180 |

$$\begin{cases} y = 40x + 130 \\ y = 40x + 180 \end{cases}$$

$$y = 40x + 130 \quad \text{Both equations are in slope-intercept form.}$$

$$y = 40x + 180 \quad \text{The lines have the same slope, but different y-intercepts.}$$

The graphs of the two equations are parallel lines, so there is no solution. If the patterns continue, sales for the two companies will never be equal.

Helpful Hint

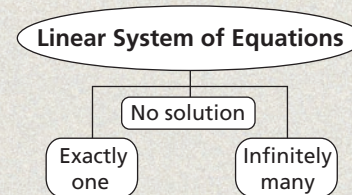
The increase in sales is the difference between sales each year.



4. Matt has \$100 in a checking account and deposits \$20 per month. Ben has \$80 in a checking account and deposits \$30 per month. Will the accounts ever have the same balance? Explain.

THINK AND DISCUSS

1. What methods can be used to determine the number of solutions of a system of linear equations?
2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write the word or words that describes a system with that number of solutions and sketch a graph.



7-1

Exercises



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Homework Help

GUIDED PRACTICE

1. **Vocabulary** A _____? _____ system can be independent or dependent. (*consistent* or *inconsistent*)

SEE EXAMPLE 1 Show that each system has no solution.

$$2. \begin{cases} y = x + 1 \\ -x + y = 3 \end{cases}$$

$$3. \begin{cases} 3x + y = 6 \\ y = -3x + 2 \end{cases}$$

$$4. \begin{cases} -y = 4x + 1 \\ 4x + y = 2 \end{cases}$$

SEE EXAMPLE 2 Show that each system has infinitely many solutions.

$$5. \begin{cases} y = -x + 3 \\ x + y - 3 = 0 \end{cases}$$

$$6. \begin{cases} y = 2x - 4 \\ 2x - y - 4 = 0 \end{cases}$$

$$7. \begin{cases} -7x + y = -2 \\ 7x - y = 2 \end{cases}$$

SEE EXAMPLE 3 Classify each system. Give the number of solutions.

$$8. \begin{cases} y = 2x + 3 \\ -2y = 2x + 6 \end{cases}$$

$$9. \begin{cases} y = -3x - 1 \\ 3x + y = 1 \end{cases}$$

$$10. \begin{cases} 9y = 3x + 18 \\ \frac{1}{3}x - y = -2 \end{cases}$$

- SEE EXAMPLE 4 11. **Athletics** Micah walks on a treadmill at 4 miles per hour. He has walked 2 miles when Luke starts running at 6 miles per hour on the treadmill next to him. If their rates continue, will Luke's distance ever equal Micah's distance? Explain.

PRACTICE AND PROBLEM SOLVING

Show that each system has no solution.

$$12. \begin{cases} y = 2x - 2 \\ -2x + y = 1 \end{cases}$$

$$13. \begin{cases} x + y = 3 \\ y = -x - 1 \end{cases}$$

$$14. \begin{cases} x + 2y = -4 \\ y = -\frac{1}{2}x - 4 \end{cases}$$

$$15. \begin{cases} -6 + y = 2x \\ y = 2x - 36 \end{cases}$$

Show that each system has infinitely many solutions.

$$16. \begin{cases} y = -2x + 3 \\ 2x + y - 3 = 0 \end{cases}$$

$$17. \begin{cases} y = x - 2 \\ x - y - 2 = 0 \end{cases}$$

$$18. \begin{cases} x + y = -4 \\ y = -x - 4 \end{cases}$$

$$19. \begin{cases} -9x - 3y = -18 \\ 3x + y = 6 \end{cases}$$

Independent Practice

| For Exercises | See Example |
|---------------|-------------|
| 12–15 | 1 |
| 16–19 | 2 |
| 20–22 | 3 |
| 23 | 4 |

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Online Extra Practice

Classify each system. Give the number of solutions.

20. $\begin{cases} y = -x + 5 \\ x + y = 5 \end{cases}$

21. $\begin{cases} y = -3x + 2 \\ y = 3x \end{cases}$

22. $\begin{cases} y - 1 = 2x \\ y = 2x - 1 \end{cases}$

23. **Sports** Mandy is skating at 5 miles per hour. Nikki is skating at 6 miles per hour and started 1 mile behind Mandy. If their rates stay the same, will Mandy catch up with Nikki? Explain.

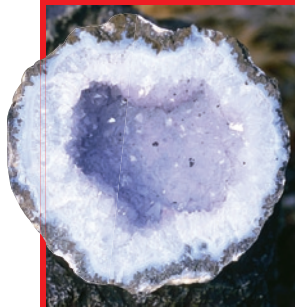
24. **Multi-Step** Photocopier A can print 35 copies per minute. Photocopier B can print 35 copies per minute. Copier B is started and makes 10 copies. Copier A is then started. If the copiers continue, will the number of copies from machine A ever equal the number of copies from machine B? Explain.

25. **Entertainment** One week Trey rented 4 DVDs and 2 video games for \$18. The next week he rented 2 DVDs and 1 video game for \$9. Find the rental costs for each video game and DVD. Explain your answer.

26. Rosa bought 1 pound of cashews and 2 pounds of peanuts for \$10. At the same store, Sabrina bought 2 pounds of cashews and 1 pound of peanuts for \$11. Find the cost per pound for cashews and peanuts.



Geology



Geodes are rounded, hollow rock formations. Most are partially or completely filled with layers of colored quartz crystals. The world's largest geode was discovered in Spain in 2000. It is 26 feet long and 5.6 feet high.

27. **Geology** Pam and Tommy collect geodes. Pam's parents gave her 2 geodes to start her collection, and she buys 4 every year. Tommy has 2 geodes that were given to him for his birthday. He buys 4 every year. If Pam and Tommy continue to buy the same amount of geodes per year, when will Tommy have as many geodes as Pam? Explain your answer.

28. Use the data given in the tables.

| | | | | |
|---|---|---|----|----|
| x | 3 | 4 | 5 | 6 |
| y | 6 | 8 | 10 | 12 |

| | | | | |
|---|----|----|----|----|
| x | 12 | 13 | 14 | 15 |
| y | 24 | 26 | 28 | 30 |

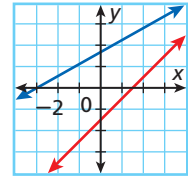
- Write an equation to describe the data in each table.
 - Graph the system of equations from part a. Describe the graph.
 - How could you have predicted the graph by looking at the equations?
 - What if...?** Each y-value in the second table increases by 1. How does this affect the graphs of the two equations? How can you tell how the graphs would be affected without actually graphing?
29. **Critical Thinking** Describe the graphs of two equations if the result of solving the system by substitution or elimination is the statement $1 = 3$.

Real-World Connections



30. The Crusader pep club is selling team buttons that support the sports teams. They contacted Buttons, Etc. which charges \$50 plus \$1.10 per button, and Logos, which charges \$40 plus \$1.10 per button.
- Write an equation for each company's cost.
 - Use the system from part a to find when the price for both companies is the same. Explain.
 - What part of the equation should the pep club negotiate to change so that the cost of Buttons, Etc. is the same as Logos? What part of the equation should change in order to get a better price?

- H.O.T.** 31. **/// ERROR ANALYSIS ///** Student A says there is no solution to the graphed system of equations. Student B says there is one solution. Which student is incorrect? Explain the error.



- H.O.T.** 32. **Write About It** Compare the graph of a system that is consistent and independent with the graph of a system that is consistent and dependent.

TEST PREP

33. Which of the following classifications fit the following system?

$$\begin{cases} 2x - y = 3 \\ 6x - 3y = 9 \end{cases}$$

- (A) Inconsistent and independent (C) Inconsistent and dependent
 (B) Consistent and independent (D) Consistent and dependent
34. Which of the following would be enough information to classify a system of two linear equations?
- (F) The graphs have the same slope.
 (G) The y -intercepts are the same.
 (H) The graphs have different slopes.
 (J) The y -intercepts are different.

CHALLENGE AND EXTEND

- H.O.T.** 35. What conditions are necessary for the system $\begin{cases} y = 2x + p \\ y = 2x + q \end{cases}$ to have infinitely many solutions? no solution?

- H.O.T.** 36. Solve the systems in parts a and b. Use this information to make a conjecture about all solutions that exist for the system in part c.

a. $\begin{cases} 3x + 4y = 0 \\ 4x + 3y = 0 \end{cases}$
 b. $\begin{cases} 2x + 5y = 0 \\ 5x + 2y = 0 \end{cases}$
 c. $\begin{cases} ax + by = 0 \\ bx + ay = 0 \end{cases}$ for $a > 0, b > 0, a \neq b$

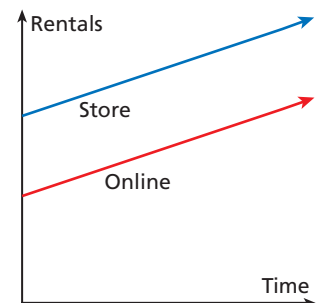
MATHEMATICAL PRACTICES

FOCUS ON MATHEMATICAL PRACTICES

- H.O.T.** 37. **Reasoning** In the graph of a linear system of equations, the lines have different slopes and the same y -intercept.

- a. Write and solve a system of equations whose graph fits this description.
 b. In general, what is the solution to a system like this? Explain.

- H.O.T.** 38. **Analysis** Chris manages a video rental store and an online video rental service. He found equations for how the number of rentals for each service changed over time and graphed them as shown. Explain what the graph means in this context.



7-2

Solving Linear Inequalities

Essential Question: How can you solve linear inequalities by using graphs?

Objective

Graph and solve linear inequalities in two variables.

Vocabulary

linear inequality
solution of a linear inequality

Who uses this?

Consumers can use linear inequalities to determine how much food they can buy for an event. (See Example 3.)

A **linear inequality** is similar to a linear equation, but the equal sign is replaced with an inequality symbol. A **solution of a linear inequality** is any ordered pair that makes the inequality true.



Animated Math

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EXAMPLE

Prep. for MCC9–12.A.REI.12

1 Identifying Solutions of Inequalities

Tell whether the ordered pair is a solution of the inequality.

A $(7, 3); y < x - 1$

$$\begin{array}{r|l} y < x - 1 \\ 3 & 7 - 1 \\ 3 & < 6 \checkmark \end{array} \quad \begin{array}{l} \text{Substitute } (7, 3) \\ \text{for } (x, y). \end{array}$$

$(7, 3)$ is a solution.

B $(4, 5); y > 3x + 2$

$$\begin{array}{r|l} y > 3x + 2 \\ 5 & 3(4) + 2 \\ 5 & 12 + 2 \\ 5 & > 14 \times \end{array} \quad \begin{array}{l} \text{Substitute } (4, 5) \\ \text{for } (x, y). \end{array}$$

$(4, 5)$ is not a solution.

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Tell whether the ordered pair is a solution of the inequality.

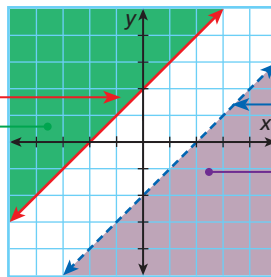
1a. $(4, 5); y < x + 1$

1b. $(1, 1); y > x - 7$

A linear inequality describes a region of a coordinate plane called a *half-plane*. All points in the region are solutions of the linear inequality. The boundary line of the region is the graph of the related equation.

When the inequality is written as $y \leq$ or $y \geq$, the points on the boundary line are solutions of the inequality, and the line is **solid**.

When the inequality is written as $y >$ or $y \geq$, the points **above** the boundary line are solutions of the inequality.



When the inequality is written as $y <$ or $y >$, the points on the boundary line are not solutions of the inequality, and the line is **dashed**.

When the inequality is written as $y <$ or $y \leq$, the points **below** the boundary line are solutions of the inequality.



Graphing Linear Inequalities

| | |
|--------|--|
| Step 1 | Solve the inequality for y . |
| Step 2 | Graph the boundary line. Use a solid line for \leq or \geq . Use a dashed line for $<$ or $>$. |
| Step 3 | Shade the half-plane above the line for $y >$ or $y \geq$. Shade the half-plane below the line for $y <$ or $y \leq$. Check your answer. |

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EXAMPLE
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2 Graphing Linear Inequalities in Two Variables

Graph the solutions of each linear inequality.

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Helpful Hint

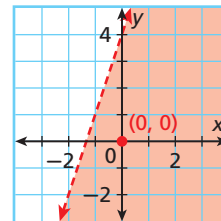
The point $(0, 0)$ is a good test point to use if it does not lie on the boundary line.

A $y < 3x + 4$

Step 1 The inequality is already solved for y .

Step 2 Graph the boundary line $y = 3x + 4$. Use a dashed line for $<$.

Step 3 The inequality is $<$, so shade below the line.



Check

| | | |
|--------------|----------------|---|
| $y < 3x + 4$ | $0 < 3(0) + 4$ | <i>Substitute $(0, 0)$ for (x, y) because it is not on the boundary line.</i> |
| | $0 < 0 + 4$ | |
| | $0 < 4$ ✓ | |

The point $(0, 0)$ satisfies the inequality, so the graph is shaded correctly.

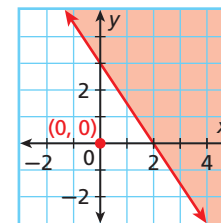
B $3x + 2y \geq 6$

Step 1 Solve the inequality for y .

$$\begin{aligned}
 3x + 2y &\geq 6 \\
 \underline{-3x} \quad \quad \underline{-3x} & \\
 2y &\geq -3x + 6 \\
 y &\geq -\frac{3}{2}x + 3
 \end{aligned}$$

Step 2 Graph the boundary line $y = -\frac{3}{2}x + 3$. Use a solid line for \geq .

Step 3 The inequality is \geq , so shade above the line.



Check

| | | |
|---------------------------|-----------------------------|--|
| $y \geq \frac{3}{2}x + 3$ | $0 \geq \frac{3}{2}(0) + 3$ | <i>A false statement means that the half-plane containing $(0, 0)$ should NOT be shaded. $(0, 0)$ is not one of the solutions, so the graph is shaded correctly.</i> |
| | $0 \geq 0 + 3$ | |
| | $0 \geq 3$ ✗ | |



Graph the solutions of each linear inequality.

2a. $4x - 3y > 12$ 2b. $2x - y - 4 > 0$ 2c. $y \geq -\frac{2}{3}x + 1$

Consumer Economics Application



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Sarah can spend at most \$7.50 on vegetables for a party. Broccoli costs \$1.25 per bunch and carrots cost \$0.75 per package.

a. Write a linear inequality to describe the situation.

Let x represent the number of bunches of broccoli and let y represent the number of packages of carrots.

Write an inequality. Use \leq for “at most.”

Cost of broccoli plus cost of carrots is at most \$7.50.

$$1.25x + 0.75y \leq 7.50$$

Solve the inequality for y .

$$1.25x + 0.75y \leq 7.50$$

$$100(1.25x + 0.75y) \leq 100(7.50)$$

$$125x + 75y \leq 750$$

$$\underline{-125x} \quad \underline{-125x}$$

$$75y \leq 750 - 125x$$

$$\underline{75y} \leq \underline{750 - 125x}$$

$$\underline{75} \quad \underline{75}$$

$$y \leq 10 - \frac{5}{3}x$$

You can multiply both sides of the inequality by 100 to eliminate the decimals.

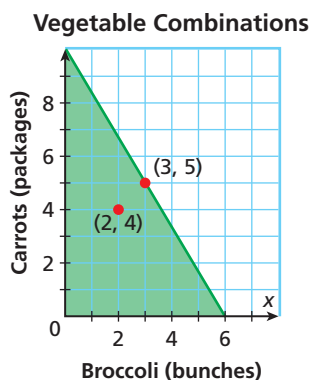
Subtraction Property of Inequality

Division Property of Inequality

b. Graph the solutions.

Step 1 Since Sarah cannot buy a negative amount of vegetables, the system is graphed only in Quadrant I. Graph the boundary line $y = -\frac{5}{3}x + 10$. Use a solid line for \leq .

Step 2 Shade below the line. Sarah must buy whole numbers of bunches or packages. All points on or below the line with whole-number coordinates represent combinations of broccoli and carrots that Sarah can buy.



c. Give two combinations of vegetables that Sarah can buy.

Two different combinations that Sarah could buy for \$7.50 or less are 2 bunches of broccoli and 4 packages of carrots, or 3 bunches of broccoli and 5 packages of carrots.



3. Dirk is going to bring two types of olives to the Honor Society induction and can spend no more than \$6. Green olives cost \$2 per pound and black olives cost \$2.50 per pound.
- Write a linear inequality to describe the situation.
 - Graph the solutions.
 - Give two combinations of olives that Dirk could buy.

Writing an Inequality from a Graph

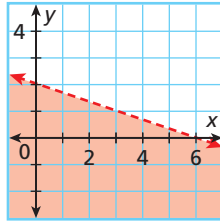
Write an inequality to represent each graph.



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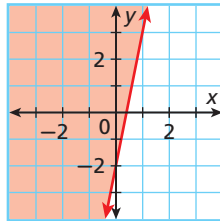


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Ay-intercept: **2**; slope: $-\frac{1}{3}$

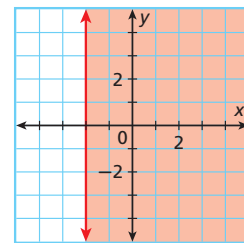
Write an equation in slope-intercept form.

$$y = mx + b \rightarrow y = -\frac{1}{3}x + 2$$

The graph is shaded *below* a *dashed* boundary line.Replace = with < to write the inequality $y < -\frac{1}{3}x + 2$.**B**y-intercept: **-2**; slope: **5**

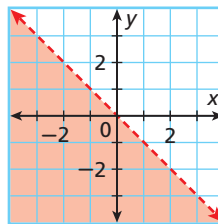
Write an equation in slope-intercept form.

$$y = mx + b \rightarrow y = 5x + (-2)$$

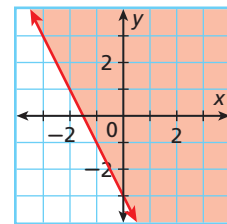
The graph is shaded *above* a *solid* boundary line.Replace = with \geq to write the inequality $y \geq 5x - 2$.**C**y-intercept: **none**; slope: **undefined**The graph is a vertical line at $x = -2$.The graph is shaded on the *right* side of a *solid* boundary line.Replace = with \geq to write the inequality $x \geq -2$.

Write an inequality to represent each graph.

4a.



4b.



MCC.MP.6

MATHEMATICAL
PRACTICES**THINK AND DISCUSS**

- Tell how graphing a linear inequality is the same as graphing a linear equation. Tell how it is different.
- Explain how you would write a linear inequality from a graph.
- GET ORGANIZED** Copy and complete the graphic organizer.



| | | | | |
|---------------|--------------|--------------|-----------------|------------------|
| Inequality | $y < 5x + 2$ | $y > 7x - 3$ | $y \leq 9x + 1$ | $y \geq -3x - 2$ |
| Symbol | < | | | |
| Boundary Line | Dashed | | | |
| Shading | Below | | | |



GUIDED PRACTICE

1. **Vocabulary** Can a *solution of a linear inequality* lie on a dashed boundary line? Explain.

SEE EXAMPLE 1 Tell whether the ordered pair is a solution of the given inequality.

2. $(0, 3); y \leq -x + 3$ 3. $(2, 0); y > -2x - 2$ 4. $(-2, 1); y < 2x + 4$

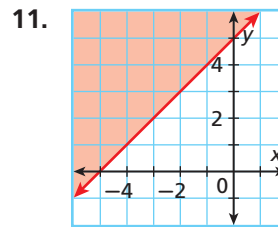
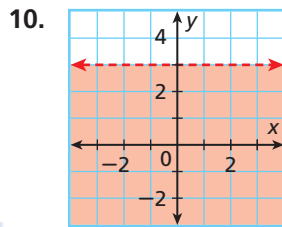
SEE EXAMPLE 2 Graph the solutions of each linear inequality.

5. $y \leq -x$ 6. $y > 3x + 1$ 7. $-y < -x + 4$ 8. $-y \geq x + 1$

SEE EXAMPLE 3 9. **Multi-Step** Jack is making punch with orange juice and pineapple juice. He can make at most 16 cups of punch.

- Write an inequality to describe the situation.
- Graph the solutions.
- Give two combinations of cups of orange juice and pineapple juice that Jack can use in his punch.

SEE EXAMPLE 4 Write an inequality to represent each graph.



PRACTICE AND PROBLEM SOLVING

Independent Practice

| For Exercises | See Example |
|---------------|-------------|
| 12–14 | 1 |
| 15–18 | 2 |
| 19 | 3 |
| 20–21 | 4 |

Tell whether the ordered pair is a solution of the given inequality.

12. $(2, 3); y \geq 2x + 3$ 13. $(1, -1); y < 3x - 3$ 14. $(0, 7); y > 4x + 7$

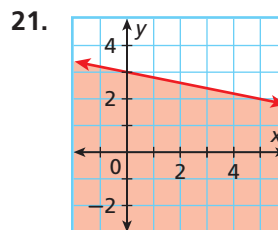
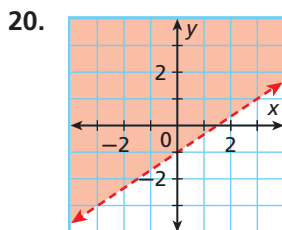
Graph the solutions of each linear inequality.

15. $y > -2x + 6$ 16. $-y \geq 2x$ 17. $x + y \leq 2$ 18. $x - y \geq 0$

19. **Multi-Step** Beverly is serving hamburgers and hot dogs at her cookout. Hamburger meat costs \$3 per pound, and hot dogs cost \$2 per pound. She wants to spend no more than \$30.

- Write an inequality to describe the situation.
- Graph the solutions.
- Give two combinations of pounds of hamburger and hot dogs that Beverly can buy.

Write an inequality to represent each graph.



22. **Business** An electronics store makes \$125 profit on every DVD player it sells and \$100 on every CD player it sells. The store owner wants to make a profit of at least \$500 a day selling DVD players and CD players.
- Write a linear inequality to determine the number of DVD players x and the number of CD players y that the owner needs to sell to meet his goal.
 - Graph the linear inequality.
 - Describe the possible values of x . Describe the possible values of y .
 - List three combinations of DVD players and CD players that the owner could sell to meet his goal.

Graph the solutions of each linear inequality.

23. $y \leq 2 - 3x$ 24. $-y < 7 + x$ 25. $2x - y \leq 4$ 26. $3x - 2y > 6$

27. **Geometry** Marvin has 18 yards of fencing that he can use to put around a rectangular garden.
- Write an inequality to describe the possible lengths and widths of the garden.
 - Graph the inequality and list three possible solutions to the problem.
 - What are the dimensions of the largest *square* garden that can be fenced in with whole-number dimensions?

28. **Hobbies** Stephen wants to buy yellow tangs and clown fish for his saltwater aquarium. He wants to spend no more than \$77 on fish. At the store, yellow tangs cost \$15 each and clown fish cost \$11 each. Write and graph a linear inequality to find the number of yellow tangs x and the number of clown fish y that Stephen could purchase. Name a solution of your inequality that is not reasonable for the situation. Explain.



Graph each inequality on a coordinate plane.

29. $y > 1$ 30. $-2 < x$ 31. $x \geq -3$ 32. $y \leq 0$
 33. $0 \geq x$ 34. $-12 + y > 0$ 35. $x + 7 < 7$ 36. $-4 \geq x - y$

37. **School** At a high school football game, tickets at the gate cost \$7 per adult and \$4 per student. Write a linear inequality to determine the number of adult and student tickets that need to be sold so that the amount of money taken in at the gate is at least \$280. Graph the inequality and list three possible solutions.

H.O.T. 38. **Critical Thinking** Why must a region of a coordinate plane be shaded to show all solutions of a linear inequality?

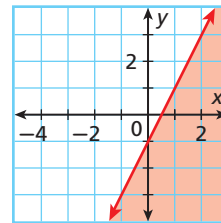
39. **Write About It** Give a real-world situation that can be described by a linear inequality. Then graph the inequality and give two solutions.

Real-World Connections

40. Gloria is making teddy bears. She is making boy and girl bears. She has enough stuffing to create 50 bears. Let x represent the number of girl bears and y represent the number of boy bears.
- Write an inequality that shows the possible number of boy and girl bears Gloria can make.
 - Graph the inequality.
 - Give three possible solutions for the numbers of boy and girl bears that can be made.

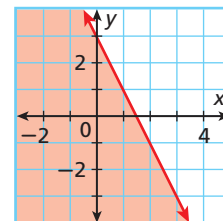


- H.O.T.** 41. **/// ERROR ANALYSIS ///** Student A wrote $y < 2x - 1$ as the inequality represented by the graph. Student B wrote $y \leq 2x - 1$ as the inequality represented by the graph. Which student is incorrect? Explain the error.
- H.O.T.** 42. **Write About It** How do you decide to shade above or below a boundary line? What does this shading represent?



TEST PREP

43. Which point is a solution of the inequality $y > -x + 3$?
- (A) (0, 3) (B) (1, 4) (C) (-1, 4) (D) (0, -3)
44. Which inequality is represented by the graph at right?
- (F) $2x + y \geq 3$ (H) $2x + y \leq 3$
 (G) $2x + y > 3$ (J) $2x + y < 3$
45. Which of the following describes the graph of $3 \leq x$?
- (A) The boundary line is dashed, and the shading is to the right.
 (B) The boundary line is dashed, and the shading is to the left.
 (C) The boundary line is solid, and the shading is to the right.
 (D) The boundary line is solid, and the shading is to the left.



CHALLENGE AND EXTEND

Graph each inequality.

46. $0 \geq -6 - 2x - 5y$ 47. $y > |x|$ 48. $y \geq |x - 3|$
49. A linear inequality has the points (0, 3) and (-3, 1.5) as solutions on the boundary line. Also, the point (1, 1) is not a solution. Write the linear inequality.
50. Two linear inequalities are graphed on the same coordinate plane. The point (0, 0) is a solution of both inequalities. The entire coordinate plane is shaded except for Quadrant I. What are the two inequalities?

MATHEMATICAL PRACTICES

FOCUS ON MATHEMATICAL PRACTICES

- H.O.T.** 51. **Analysis** Equivalent inequalities have the same boundaries and contain the same points in their solutions. Is the inequality $x - y < 12$ equivalent to $y < x - 12$? Explain why or why not.
- H.O.T.** 52. **Modeling** Angie volunteers at the local animal sanctuary. They are fencing a space that will house no more than 10 dogs. They want to house some adult dogs x and some puppies y .
- Write an inequality that describes the situation.
 - Graph the inequality. How is your graphed inequality different than the actual situation?
- H.O.T.** 53. **Make a Conjecture** The solutions to an inequality of the form $y > ax + b$ are always found above the boundary line. Where do you think the solutions to an inequality of the form $x > ay + b$ can be found? Explain your reasoning.

7-3

Solving Systems of Linear Inequalities

Essential Question: How can you solve systems of linear inequalities by using graphs?

Objective

Graph and solve systems of linear inequalities in two variables.

Vocabulary

system of linear inequalities
solutions of a system of linear inequalities

Who uses this?

The owner of a surf shop can use systems of linear inequalities to determine how many surfboards and wakeboards need to be sold to make a certain profit. (See Example 4.)



A **system of linear inequalities** is a set of two or more linear inequalities containing two or more variables. The **solutions of a system of linear inequalities** are all of the ordered pairs that satisfy all the linear inequalities in the system.

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Prep. for MCC9-12.A.REI.12

EXAMPLE 1

Identifying Solutions of Systems of Linear Inequalities

Tell whether the ordered pair is a solution of the given system.

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A $(2, 1); \begin{cases} y < -x + 4 \\ y \leq x + 1 \end{cases}$

$$\begin{array}{r|l} (2, 1) & \\ \hline y < -x + 4 & \\ 1 & | -2 + 4 \\ \hline 1 & < 2 \quad \checkmark \end{array}$$

$$\begin{array}{r|l} (2, 1) & \\ \hline y \leq x + 1 & \\ 1 & | 2 + 1 \\ \hline 1 & \leq 3 \quad \checkmark \end{array}$$

$(2, 1)$ is a solution to the system because it satisfies both inequalities.

B $(2, 0); \begin{cases} y \geq 2x \\ y < x + 1 \end{cases}$

$$\begin{array}{r|l} (2, 0) & \\ \hline y \geq 2x & \\ 0 & | 2(2) \\ \hline 0 & \geq 4 \quad \times \end{array}$$

$$\begin{array}{r|l} (2, 0) & \\ \hline y < x + 1 & \\ 0 & | 2 + 1 \\ \hline 0 & < 3 \quad \checkmark \end{array}$$

$(2, 0)$ is not a solution to the system because it does not satisfy both inequalities.

Remember!

An ordered pair must be a solution of all inequalities to be a solution of the system.



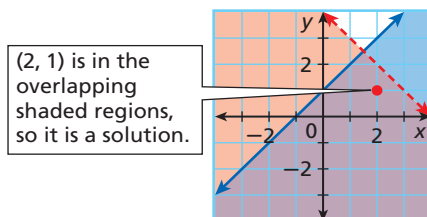
Tell whether the ordered pair is a solution of the given system.

1a. $(0, 1); \begin{cases} y < -3x + 2 \\ y \geq x - 1 \end{cases}$

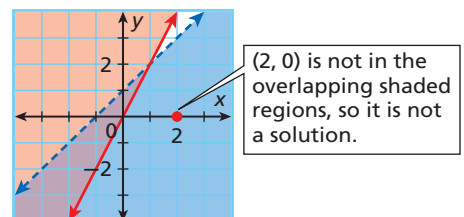
1b. $(0, 0); \begin{cases} y > -x + 1 \\ y > x - 1 \end{cases}$

To show all the solutions of a system of linear inequalities, graph the solutions of each inequality. The solutions of the system are represented by the overlapping shaded regions. Below are graphs of Examples 1A and 1B.

Example 1A



Example 1B



Solving a System of Linear Inequalities by Graphing

Graph the system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

$$\begin{cases} 8x + 4y \leq 12 \\ y > \frac{1}{2}x - 2 \end{cases}$$

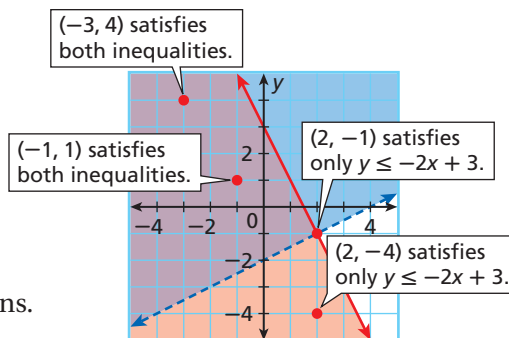
$$\begin{aligned} 8x + 4y &\leq 12 \\ 4y &\leq -8x + 12 \\ y &\leq -2x + 3 \end{aligned}$$

Solve the first inequality for y .

Graph the system.

$$\begin{cases} y \leq -2x + 3 \\ y > \frac{1}{2}x - 2 \end{cases}$$

$(-1, 1)$ and $(-3, 4)$ are solutions.
 $(2, -1)$ and $(2, -4)$ are not solutions.



Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

2a. $\begin{cases} y \leq x + 1 \\ y > 2 \end{cases}$

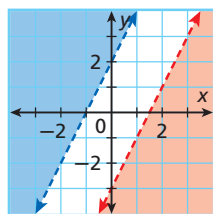
2b. $\begin{cases} y > x - 7 \\ 3x + 6y \leq 12 \end{cases}$

Previously, you saw that in systems of linear equations, if the lines are parallel, there are no solutions. With systems of linear inequalities, that is not always true.

Graphing Systems with Parallel Boundary Lines

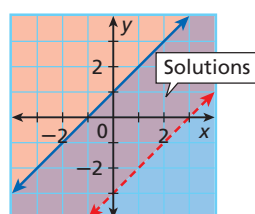
Graph each system of linear inequalities. Describe the solutions.

A $\begin{cases} y < 2x - 3 \\ y > 2x + 2 \end{cases}$



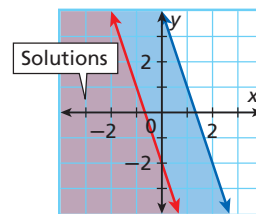
This system has no solution.

B $\begin{cases} y > x - 3 \\ y \leq x + 1 \end{cases}$



The solutions are all points between the parallel lines and on the solid line.

C $\begin{cases} y \leq -3x - 2 \\ y \leq -3x + 4 \end{cases}$



The solutions are the same as the solutions of $y \leq -3x - 2$.

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Graph each system of linear inequalities. Describe the solutions.

3a. $\begin{cases} y > x + 1 \\ y \leq x - 3 \end{cases}$

3b. $\begin{cases} y \geq 4x - 2 \\ y \leq 4x + 2 \end{cases}$

3c. $\begin{cases} y > -2x + 3 \\ y > -2x \end{cases}$

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EXAMPLE

MCC9-12.A.CED.3

4 Business Application

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A surf shop makes the profits given in the table. The shop owner sells at least 10 surfboards and at least 20 wakeboards per month. He wants to earn at least \$2000 a month. Show and describe all possible combinations of surfboards and wakeboards that the store owner needs to sell to meet his goals. List two possible combinations.

| Profit per Board Sold (\$) | |
|----------------------------|-----|
| Surfboard | 150 |
| Wakeboard | 100 |

Step 1 Write a system of inequalities.

Let x represent the number of surfboards and y represent the number of wakeboards.

$x \geq 10$

He sells at least 10 surfboards.

$y \geq 20$

He sells at least 20 wakeboards.

$150x + 100y \geq 2000$

He wants to earn a total of at least \$2000.

Step 2 Graph the system.

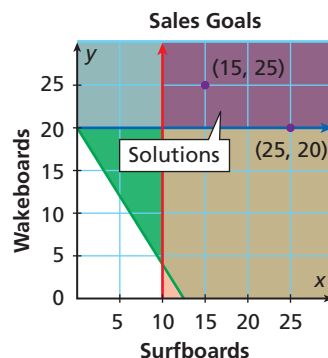
The graph should be in only the first quadrant because sales are not negative.

Step 3 Describe all possible combinations.

To meet the sales goals, the shop could sell any combination represented by an ordered pair of whole numbers in the solution region. Answers must be whole numbers because the shop cannot sell part of a surfboard or wakeboard.

Step 4 List two possible combinations.

- Two possible combinations are:
- 15 surfboards and 25 wakeboards
- 25 surfboards and 20 wakeboards



Caution!

An ordered pair solution of the system need not have whole numbers, but answers to many application problems may be restricted to whole numbers.



4. At her party, Alice is serving pepper jack cheese and cheddar cheese. She wants to have at least 2 pounds of each. Alice wants to spend at most \$20 on cheese. Show and describe all possible combinations of the two cheeses Alice could buy. List two possible combinations.

| Price per Pound (\$) | |
|----------------------|---|
| Pepper Jack | 4 |
| Cheddar | 2 |

MCC.MP.1

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. How would you write a system of linear inequalities from a graph?

2. **GET ORGANIZED** Copy and complete each part of the graphic organizer. In each box, draw a graph and list one solution.



$\begin{cases} y \geq 2x + 1 \\ y > \frac{1}{2}x - 2 \end{cases}$

$\begin{cases} y < 2x + 1 \\ y \geq \frac{1}{2}x - 2 \end{cases}$

Graph

Solution

Graph

Solution



GUIDED PRACTICE

1. **Vocabulary** A solution of a system of inequalities is a solution of _____? _____ of the inequalities in the system. (*at least one* or *all*)

SEE EXAMPLE 1 Tell whether the ordered pair is a solution of the given system.

2. $(0, 0); \begin{cases} y < -x + 3 \\ y < x + 2 \end{cases}$ 3. $(0, 0); \begin{cases} y < 3 \\ y > x - 2 \end{cases}$ 4. $(1, 0); \begin{cases} y > 3x \\ y \leq x + 1 \end{cases}$

SEE EXAMPLE 2 Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

5. $\begin{cases} y < 2x - 1 \\ y > 2 \end{cases}$ 6. $\begin{cases} x < 3 \\ y > x - 2 \end{cases}$ 7. $\begin{cases} y \geq 3x \\ 3x + y \geq 3 \end{cases}$ 8. $\begin{cases} 2x - 4y \leq 8 \\ y > x - 2 \end{cases}$

SEE EXAMPLE 3 Graph each system of linear inequalities. Describe the solutions.

9. $\begin{cases} y > 2x + 3 \\ y < 2x \end{cases}$ 10. $\begin{cases} y \leq -3x - 1 \\ y \geq -3x + 1 \end{cases}$ 11. $\begin{cases} y > 4x - 1 \\ y \leq 4x + 1 \end{cases}$
 12. $\begin{cases} y < -x + 3 \\ y > -x + 2 \end{cases}$ 13. $\begin{cases} y > 2x - 1 \\ y > 2x - 4 \end{cases}$ 14. $\begin{cases} y \leq -3x + 4 \\ y \leq -3x - 3 \end{cases}$

SEE EXAMPLE 4 15. **Business** Sandy makes \$2 profit on every cup of lemonade that she sells and \$1 on every cupcake that she sells. Sandy wants to sell at least 5 cups of lemonade and at least 5 cupcakes per day. She wants to earn at least \$25 per day. Show and describe all the possible combinations of lemonade and cupcakes that Sandy needs to sell to meet her goals. List two possible combinations.

PRACTICE AND PROBLEM SOLVING

Independent Practice

| For Exercises | See Example |
|---------------|-------------|
| 16–18 | 1 |
| 19–22 | 2 |
| 23–28 | 3 |
| 29 | 4 |

Tell whether the ordered pair is a solution of the given system.

16. $(0, 0); \begin{cases} y > -x - 1 \\ y < 2x + 4 \end{cases}$ 17. $(0, 0); \begin{cases} x + y < 3 \\ y > 3x - 4 \end{cases}$ 18. $(1, 0); \begin{cases} y > 3x \\ y > 3x + 1 \end{cases}$

Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

19. $\begin{cases} y < -3x - 3 \\ y \geq 0 \end{cases}$ 20. $\begin{cases} y < -1 \\ y > 2x - 1 \end{cases}$ 21. $\begin{cases} y > 2x + 4 \\ 6x + 2y \geq -2 \end{cases}$ 22. $\begin{cases} 9x + 3y \leq 6 \\ y > x \end{cases}$

Graph each system of linear inequalities. Describe the solutions.

23. $\begin{cases} y < 3 \\ y > 5 \end{cases}$ 24. $\begin{cases} y < x - 1 \\ y > x - 2 \end{cases}$ 25. $\begin{cases} x \geq 2 \\ x \leq 2 \end{cases}$
 26. $\begin{cases} y > -4x - 3 \\ y < -4x + 2 \end{cases}$ 27. $\begin{cases} y > -1 \\ y > 2 \end{cases}$ 28. $\begin{cases} y \leq 2x + 1 \\ y \leq 2x - 4 \end{cases}$



29. **Multi-Step** Linda works at a pharmacy for \$15 an hour. She also baby-sits for \$10 an hour. Linda needs to earn at least \$90 per week, but she does not want to work more than 20 hours per week. Show and describe the number of hours Linda could work at each job to meet her goals. List two possible solutions.
30. **Farming** Tony wants to plant at least 40 acres of corn and at least 50 acres of soybeans. He wants no more than 200 acres of corn and soybeans. Show and describe all the possible combinations of the number of acres of corn and of soybeans Tony could plant. List two possible combinations.

Graph each system of linear inequalities.

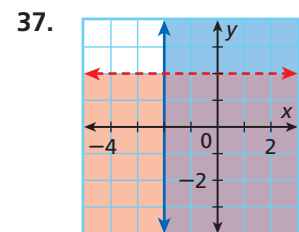
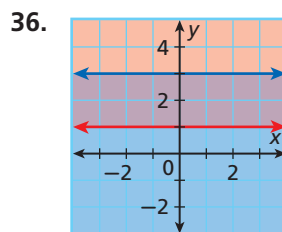
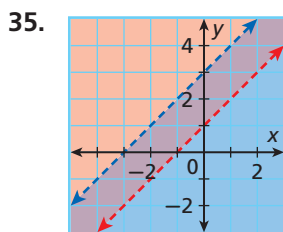
31.
$$\begin{cases} y \geq -3 \\ y \geq 2 \end{cases}$$

32.
$$\begin{cases} y > -2x - 1 \\ y > -2x - 3 \end{cases}$$

33.
$$\begin{cases} x \leq -3 \\ x \geq 1 \end{cases}$$

34.
$$\begin{cases} y < 4 \\ y > 0 \end{cases}$$

Write a system of linear inequalities to represent each graph.

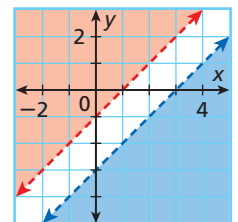


38. **Military** For males to enter the United States Air Force Academy, located in Colorado Springs, CO, they must be at least 17 but less than 23 years of age. Their standing height must be not less than 60 inches and not greater than 80 inches. Graph all possible heights and ages for eligible male candidates. Give three possible combinations.

39. **ERROR ANALYSIS** Two students wrote a system of linear inequalities to describe the graph. Which student is incorrect? Explain the error.

A
$$\begin{cases} y < x - 3 \\ y > x - 1 \end{cases}$$

B
$$\begin{cases} y > x - 3 \\ y < x - 1 \end{cases}$$



40. **Recreation** Vance wants to fence in a rectangular area for his dog. He wants the length of the rectangle to be at least 30 feet and the perimeter to be no more than 150 feet. Graph all possible dimensions of the rectangle.

- HOT:** 41. **Critical Thinking** Can the solutions of a system of linear inequalities be the points on a line? Explain.

Real-World Connections



42. Gloria is starting her own company making teddy bears. She has enough bear bodies to create 40 bears. She will make girl bears and boy bears.
- Write an inequality to show this situation.
 - Gloria will charge \$15 for girl bears and \$12 for boy bears. She wants to earn at least \$540 a week. Write an inequality to describe this situation.
 - Graph this situation and locate the solution region.

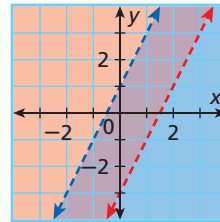
- H.O.T.** 43. **Write About It** What must be true of the boundary lines in a system of two linear inequalities if there is no solution of the system? Explain.

TEST PREP

44. Which point is a solution of $\begin{cases} 2x + y \geq 3 \\ y \geq -2x + 1 \end{cases}$?
- (A) (0, 0) (B) (0, 1) (C) (1, 0) (D) (1, 1)

45. Which system of inequalities best describes the graph?

- (F) $\begin{cases} y < 2x - 3 \\ y > 2x + 1 \end{cases}$ (H) $\begin{cases} y < 2x - 3 \\ y < 2x + 1 \end{cases}$
- (G) $\begin{cases} y > 2x - 3 \\ y < 2x + 1 \end{cases}$ (J) $\begin{cases} y > 2x - 3 \\ y > 2x + 1 \end{cases}$



46. **Short Response** Graph and describe $\begin{cases} y + x > 2 \\ y \leq -3x + 4 \end{cases}$. Give two possible solutions of the system.

CHALLENGE AND EXTEND

47. **Estimation** Graph the given system of inequalities. Estimate the area of the overlapping solution regions.

$$\begin{cases} y \geq 0 \\ y \leq x + 3.5 \\ y \leq -x + 3.5 \end{cases}$$

48. Write a system of linear inequalities for which $(-1, 1)$ and $(1, 4)$ are solutions and $(0, 0)$ and $(2, -1)$ are not solutions.
49. Graph $|y| < 1$.
50. Write a system of linear inequalities for which the solutions are all the points in the third quadrant.

MATHEMATICAL PRACTICES

FOCUS ON MATHEMATICAL PRACTICES

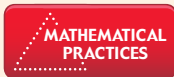
- H.O.T.** 51. **Problem Solving** Without graphing, describe the solution or solutions to the system of inequalities shown. How did you find your answer? $\begin{cases} x - y < 3 \\ x - y > 3 \end{cases}$
- H.O.T.** 52. **Analysis** Is it possible for a system of two linear inequalities to have a single point as a solution? What about a system of more than two inequalities? If either case is possible, write such a system and name the solution.
- H.O.T.** 53. **Reasoning** Use a graph to find three solutions to the system of inequalities shown. What approach did you use to graph the system? $\begin{cases} y < x^2 + 5 \\ y > x^2 \end{cases}$

7-3 Technology TASK

Solve Systems of Linear Inequalities

A graphing calculator gives a visual solution to a system of linear inequalities.

Use with Solving Systems of Linear Inequalities



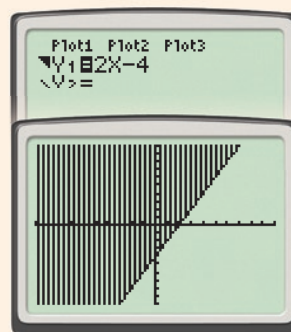
Use appropriate tools strategically.

MCC9-12.A.REI.12 Graph the ... solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Activity

Graph the system $\begin{cases} y > 2x - 4 \\ 2.75y - x < 6 \end{cases}$. Give two ordered pairs that are solutions.

- The first inequality is solved for y .
- Graph the first inequality. First graph the boundary line $y = 2x - 4$. Press **Y=** and enter $2x - 4$ for **Y1**.
The inequality contains the symbol $>$. The solution region is above the boundary line. Press **2nd** to move the cursor to the left of **Y1**. Press **ENTER** until the icon that looks like a region above a line appears. Press **GRAPH**.



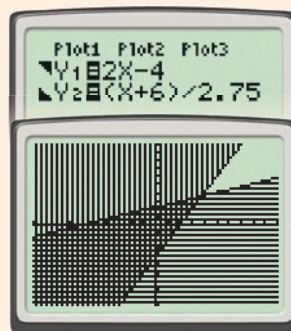
- Solve the second inequality for y .

$$2.75y - x < 6$$

$$2.75y < x + 6$$

$$y < \frac{x + 6}{2.75}$$

- Graph the second inequality. First graph the boundary line $y = \frac{x + 6}{2.75}$. Press **Y=** and enter $(x + 6)/2.75$ for **Y2**.
The inequality contains the symbol $<$. The solution region is below the boundary line. Press **2nd** to move the cursor to the left of **Y2**. Press **ENTER** until the icon that looks like a region below a line appears. Press **GRAPH**.



- The solutions of the system are represented by the overlapping shaded regions. The points $(0, 0)$ and $(-1, 0)$ are in the shaded region.

Check Test $(0, 0)$ in both inequalities.

$$\begin{array}{l|l} y > 2x - 4 & 2.75y - x < 6 \\ \hline 0 > 2(0) - 4 & 2.75(0) - 0 < 6 \\ 0 > -4 \checkmark & 0 < 6 \checkmark \end{array}$$

Test $(-1, 0)$ in both inequalities.

$$\begin{array}{l|l} y > 2x - 4 & 2.75y - x < 6 \\ \hline 0 > 2(-1) - 4 & 2.75(0) - (-1) < 6 \\ 0 > -6 \checkmark & 1 < 6 \checkmark \end{array}$$

Try This

Graph each system. Give two ordered pairs that are solutions.

1. $\begin{cases} x + 5y > -10 \\ x - y < 4 \end{cases}$

2. $\begin{cases} y > x - 2 \\ y \leq x + 2 \end{cases}$

3. $\begin{cases} y > x - 2 \\ y \leq 3 \end{cases}$

4. $\begin{cases} y < x - 3 \\ y - 3 > x \end{cases}$

Ready to Go On?



7-1 Solving Special Systems

Solve each system of linear equations.

1. $\begin{cases} y = -2x - 6 \\ 2x + y = 5 \end{cases}$

2. $\begin{cases} x + y = 2 \\ 2x + 2y = -6 \end{cases}$

3. $\begin{cases} y = -2x + 4 \\ 2x + y = 4 \end{cases}$

Classify each system. Give the number of solutions.

4. $\begin{cases} 3x = -6y + 3 \\ 2y = -x + 1 \end{cases}$

5. $\begin{cases} y = -4x + 2 \\ 4x + y = -2 \end{cases}$

6. $\begin{cases} 4x - 3y = 8 \\ y = 4(x + 2) \end{cases}$

7-2 Solving Linear Inequalities

Tell whether the ordered pair is a solution of the inequality.

7. $(3, -2); y < -2x + 1$

8. $(2, 1); y \geq 3x - 5$

9. $(1, -6); y \leq 4x - 10$

Graph the solutions of each linear inequality.

10. $y \geq 4x - 3$

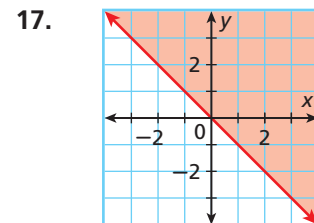
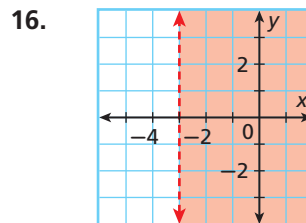
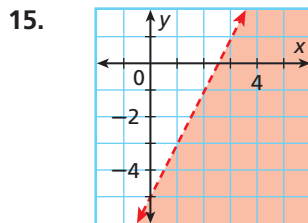
11. $3x - y < 5$

12. $2x + 3y < 9$

13. $y \leq -\frac{1}{2}x$

14. Theo's mother has given him at most \$150 to buy clothes for school. The pants cost \$30 each and the shirts cost \$15 each. Write a linear inequality to describe the situation. Graph the solutions and give three combinations of pants and shirts that Theo could buy.

Write an inequality to represent each graph.



7-3 Solving Systems of Linear Inequalities

Tell whether the ordered pair is a solution of the given system.

18. $(-3, -1); \begin{cases} y > -2 \\ y < x + 4 \end{cases}$

19. $(-3, 0); \begin{cases} y \leq x + 4 \\ y \geq -2x - 6 \end{cases}$

20. $(0, 0); \begin{cases} y \geq 3x \\ 2x + y < -1 \end{cases}$

Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

21. $\begin{cases} y > -2 \\ y < x + 3 \end{cases}$

22. $\begin{cases} x + y \leq 2 \\ 2x + y \geq -1 \end{cases}$

23. $\begin{cases} 2x - 5y \leq -5 \\ 3x + 2y < 10 \end{cases}$

Graph each system of linear inequalities. Describe the solutions.

24.
$$\begin{cases} y \geq x + 1 \\ y \geq x - 4 \end{cases}$$

25.
$$\begin{cases} y \geq 2x - 1 \\ y < 2x - 3 \end{cases}$$

26.
$$\begin{cases} y < -3x + 5 \\ y > -3x - 2 \end{cases}$$

27. A grocer sells mangos for \$4/lb and apples for \$3/lb. The grocer starts with 45 lb of mangos and 50 lb of apples each day. The grocer's goal is to make at least \$300 by selling mangos and apples each day. Show and describe all possible combinations of mangos and apples that could be sold to meet the goal. List two possible combinations.

PARCC Assessment Readiness

COMMON
CORE
GPS

Selected Response

1. Elena and her husband Marc both drive to work. Elena's car has a current mileage (total distance driven) of 5,000 and she drives 15,000 miles more each year. Marc's car has a current mileage of 32,000 and he drives 15,000 miles more each year. Will the mileages for the two cars ever be equal? Explain.

- (A) No; The equations have different slopes, so the lines do not intersect.
 (B) Yes; The equations have different y -intercepts, so the lines intersect.
 (C) No; the equations have equal slopes but different y -intercepts, so the lines do not intersect.
 (D) Yes; The equations have different slopes, so the lines intersect.

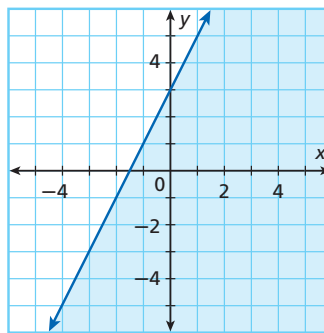
2. Classify $\begin{cases} x - 8y = 6 \\ 2x - 16y = 12 \end{cases}$. Give the number of solutions.

- (F) This system is consistent. It has infinitely many solutions.
 (G) This system is inconsistent. It has infinitely many solutions.
 (H) This system is inconsistent. It has no solutions.
 (J) This system is consistent. It has one solution.

3. Solve $\begin{cases} y = -x + 8 \\ x + y = 7 \end{cases}$

- (A) This system has infinitely many solutions.
 (B) This system has no solutions.
 (C) $(\frac{1}{2}, \frac{15}{2})$
 (D) $(-\frac{1}{2}, \frac{17}{2})$

4. Write an inequality to represent the graph.



- (F) $y > 2x + 3$ (H) $y < 3x + 2$
 (G) $y \leq 2x + 3$ (J) $y < 2x + 3$

Mini-Task

5. Graph the system of linear inequalities $\begin{cases} y < -3x + 2 \\ y \geq 4x - 1 \end{cases}$.

Give two ordered pairs that are solutions and two that are not solutions.