

# Solving Multi-Step Inequalities



MCC9-12.A.REI.3

MCC9-12.A.REI.3

MCC9-12.A.CED.1

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The Common Core Georgia Performance Standards for Mathematical Practice describe varieties of expertise that all students should seek to develop.

Opportunities to develop these practices are integrated throughout this program.

- |   |   |
|---|---|
| <b>1</b> Make sense of problems and persevere in solving them.            | <b>5</b> Use appropriate tools strategically.                   |
| <b>2</b> Reason abstractly and quantitatively.                            | <b>6</b> Attend to precision.                                   |
| <b>3</b> Construct viable arguments and critique the reasoning of others. | <b>7</b> Look for and make use of structure.                    |
| <b>4</b> Model with mathematics.  | <b>8</b> Look for and express regularity in repeated reasoning. |

# Unpacking the Standards



Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this chapter.



## MCC9-12.A.CED.1

Create ... inequalities ... in one variable and use them to solve problems.



### What It Means For You

You can write an inequality to represent a real-world problem and then solve the inequality to find the possible answers.

### EXAMPLE

Amy uses  $\frac{3}{4}$  cup of vanilla yogurt to make a smoothie. What are the possible whole numbers of smoothies that Amy can make using 1 quart of vanilla yogurt?

Let  $s$  represent the number of smoothies Amy can make.

cups per smoothie	•	number of smoothies	≤	cups per quart
$\frac{3}{4}$	•	$s$	≤	4
		$\frac{3}{4}s$	≤	4
		$s$	≤	$\frac{16}{3}$

Amy can make 0, 1, 2, 3, 4, or 5 smoothies.



## MCC9-12.A.REI.3

Solve linear ... inequalities in one variable, ...

### Key Vocabulary

#### linear inequality in one variable

(*desigualdad lineal en una variable*)

An inequality that can be written in one of the following forms:  $ax < b$ ,  $ax > b$ ,  $ax \leq b$ ,  $ax \geq b$ , or  $ax \neq b$ , where  $a$  and  $b$  are constants and  $a \neq 0$ .

### What It Means For You

Solving inequalities lets you answer questions where a range of solutions is possible.

### EXAMPLE

Solve the inequality for  $t$  to find what grades on the final exam will give Cleo a course grade of "A".

$$705 + 2t \geq 895 \quad \text{Cleo has 705 points and needs at least 895.}$$

$$2t \geq 190 \quad \text{Subtract 705 from both sides.}$$

$$t \geq 95 \quad \text{Divide both sides by 2.}$$

Cleo needs to earn a 95 or above on the final exam.

# 5-1

## Solving Two-Step and Multi-Step Inequalities

**Essential Question:** How can you solve inequalities that involve more than one operation?

### Objective

Solve inequalities that contain more than one operation.

### Who uses this?

Contestants at a county fair can solve an inequality to find how many pounds a prize-winning pumpkin must weigh. (See Example 3.)

At the county fair, contestants can enter contests that judge animals, recipes, crops, art projects, and more. Sometimes an average score or average weight is used to determine the winner of the blue ribbon. A contestant can use a multi-step inequality to determine what score or weight is needed in order to win.

Inequalities that contain more than one operation require more than one step to solve. Use inverse operations to undo the operations in the inequality one at a time.



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### EXAMPLE 1

MCC9-12.A.REI.3

### Solving Multi-Step Inequalities

Solve each inequality and graph the solutions.

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**A**  $160 + 4f \leq 500$

$$160 + 4f \leq 500$$

$$\underline{-160} \quad \underline{-160}$$

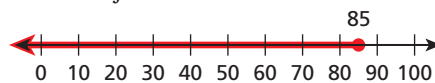
$$4f \leq 340$$

$$\underline{\frac{4f}{4}} \leq \underline{\frac{340}{4}}$$

$$f \leq 85$$

Since 160 is added to  $4f$ , subtract 160 from both sides to undo the addition.

Since  $f$  is multiplied by 4, divide both sides by 4 to undo the multiplication.



**B**  $7 - 2t \leq 21$

$$7 - 2t \leq 21$$

$$\underline{-7} \quad \underline{-7}$$

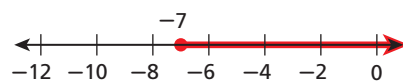
$$-2t \leq 14$$

$$\underline{\frac{-2t}{-2}} \geq \underline{\frac{14}{-2}}$$

$$t \geq -7$$

Since 7 is added to  $-2t$ , subtract 7 from both sides to undo the addition.

Since  $t$  is multiplied by  $-2$ , divide both sides by  $-2$  to undo the multiplication. Change  $\leq$  to  $\geq$ .



Solve each inequality and graph the solutions.

1a.  $-12 \geq 3x + 6$

1b.  $\frac{x+5}{-2} > 3$

1c.  $\frac{1-2n}{3} \geq 7$

To solve more complicated inequalities, you may first need to simplify the expressions on one or both sides by using the order of operations, combining like terms, or using the Distributive Property.

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**EXAMPLE**  
MCC9-12.A.REI.3

**2**

## Simplifying Before Solving Inequalities

Solve each inequality and graph the solutions.

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**A**  $-4 + (-8) < -5c - 2$

$$-12 < -5c - 2$$

$$\begin{array}{r} +2 \quad +2 \\ -12 < -5c - 2 \end{array}$$

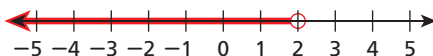
$$-10 < -5c$$

$$\begin{array}{r} -10 \\ -5 \end{array} > \begin{array}{r} -5c \\ -5 \end{array}$$

$$2 > c \text{ (or } c < 2)$$

Combine like terms. Since 2 is subtracted from  $-5c$ , add 2 to both sides to undo the subtraction.

Since  $c$  is multiplied by  $-5$ , divide both sides by  $-5$  to undo the multiplication. Change  $<$  to  $>$ .



**B**  $-3(3 - x) < 4^2$

$$-3(3 - x) < 4^2$$

$$-3(3) - (-3)x < 4^2$$

$$-9 + 3x < 4^2$$

$$-9 + 3x < 16$$

$$-9 + 3x < 16$$

$$\begin{array}{r} +9 \quad +9 \\ -9 + 3x < 16 \end{array}$$

$$3x < 25$$

$$\begin{array}{r} 3x \\ 3 \end{array} < \begin{array}{r} 25 \\ 3 \end{array}$$

$$x < 8\frac{1}{3}$$

Distribute  $-3$  on the left side.

Simplify the right side.

Since  $-9$  is added to  $3x$ , add 9 to both sides to undo the addition.

Since  $x$  is multiplied by 3, divide both sides by 3 to undo the multiplication.



**C**  $\frac{4}{5}x + \frac{1}{2} > \frac{3}{5}$

$$10\left(\frac{4}{5}x + \frac{1}{2}\right) > 10\left(\frac{3}{5}\right)$$

$$10\left(\frac{4}{5}x\right) + 10\left(\frac{1}{2}\right) > 10\left(\frac{3}{5}\right)$$

$$8x + 5 > 6$$

$$\begin{array}{r} -5 \quad -5 \\ 8x + 5 > 6 \end{array}$$

$$8x > 1$$

$$\begin{array}{r} 8x \\ 8 \end{array} > \begin{array}{r} 1 \\ 8 \end{array}$$

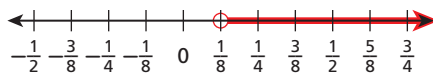
$$x > \frac{1}{8}$$

Multiply both sides by 10, the LCD of the fractions.

Distribute 10 on the left side.

Since 5 is added to  $8x$ , subtract 5 from both sides to undo the addition.

Since  $x$  is multiplied by 8, divide both sides by 8 to undo the multiplication.



Solve each inequality and graph the solutions.

2a.  $2m + 5 > 5^2$

2b.  $3 + 2(x + 4) > 3$

2c.  $\frac{5}{8} < \frac{3}{8}x - \frac{1}{4}$

## Gardening Application



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To win the blue ribbon for the Heaviest Pumpkin Crop at the county fair, the average weight of John's two pumpkins must be greater than 819 lb. One of his pumpkins weighs 887 lb. What is the least number of pounds the second pumpkin could weigh in order for John to win the blue ribbon?



(887 plus  $p$ ) divided by 2 must be greater than 819.

(887 +  $p$ ) ÷ 2 > 819

$$\frac{887 + p}{2} > 819$$

Since  $887 + p$  is divided by 2, multiply both sides by 2 to undo the division.

$$2\left(\frac{887 + p}{2}\right) > 2(819)$$

$$887 + p > 1638$$

Since 887 is added to  $p$ , subtract 887 from both sides to undo the addition.

$$\begin{array}{r} -887 \\ 887 + p > 1638 \\ \hline p > 751 \end{array}$$

The second pumpkin must weigh more than 751 pounds.

**Check** Check the endpoint, 751.

Check a number greater than 751.

$$\begin{array}{r} \frac{887 + p}{2} = 819 \\ \hline \frac{887 + 751}{2} \quad | \quad 819 \\ \frac{1638}{2} \quad | \quad 819 \\ 819 \quad | \quad 819 \checkmark \end{array}$$

$$\begin{array}{r} \frac{887 + p}{2} > 819 \\ \hline \frac{887 + 755}{2} > 819 \\ \frac{1642}{2} > 819 \\ 821 > 819 \checkmark \end{array}$$



3. The average of Jim's two test scores must be at least 90 to make an A in the class. Jim got a 95 on his first test. What scores can Jim get on his second test to make an A in the class?

MCC.MP.1, MCC.MP.7

MATHEMATICAL PRACTICES

## THINK AND DISCUSS

- The inequality  $v \geq 25$  states that 25 is the \_\_\_\_? \_\_\_\_\_. (*value of  $v$ , minimum value of  $v$ , or maximum value of  $v$* )
- Describe two sets of steps for solving the inequality  $\frac{x+5}{3} > 7$ .
- GET ORGANIZED** Copy and complete the graphic organizer.

Know it!

Note

Solving Multi-Step Equations and Inequalities

How are they alike?

How are they different?



## GUIDED PRACTICE

Solve each inequality and graph the solutions.

SEE EXAMPLE 1

1.  $2m + 1 > 13$

2.  $2d + 21 \leq 11$

3.  $6 \leq -2x + 2$

4.  $4c - 7 > 5$

5.  $\frac{4+x}{3} > -4$

6.  $1 < 0.2x - 0.7$

7.  $\frac{3-2x}{3} \leq 7$

8.  $2x + 5 \geq 2$

SEE EXAMPLE 2

9.  $4(x+2) > 6$

10.  $\frac{1}{4}x + \frac{2}{3} < \frac{3}{4}$

11.  $4 - x + 6^2 \geq 21$

12.  $4 - x > 3(4 - 2)$

13.  $0.2(x - 10) > -1.8$

14.  $3(j + 41) \leq 35$

SEE EXAMPLE 3

15. **Business** A sales representative is given a choice of two paycheck plans. One choice includes a monthly base pay of \$300 plus 10% commission on his sales. The second choice is a monthly salary of \$1200. For what amount of sales would the representative make more money with the first plan?

## PRACTICE AND PROBLEM SOLVING

Solve each inequality and graph the solutions.

## Independent Practice

For Exercises	See Example
16–27	1
28–36	2
37	3



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Online Extra Practice

16.  $4r - 9 > 7$

17.  $3 \leq 5 - 2x$

18.  $\frac{w+3}{2} > 6$

19.  $11w + 99 < 77$

20.  $9 \geq \frac{1}{2}v + 3$

21.  $-4x - 8 > 16$

22.  $8 - \frac{2}{3}z \leq 2$

23.  $f + 2\frac{1}{2} < -2$

24.  $\frac{3n-8}{5} \geq 2$

25.  $-5 > -5 - 3w$

26.  $10 > \frac{5-3p}{2}$

27.  $2v + 1 > 2\frac{1}{3}$

28.  $4(x+3) > -24$

29.  $4 > x - 3(x+2)$

30.  $-18 \geq 33 - 3h$

31.  $-2 > 7x - 2(x-4)$

32.  $9 - (9)^2 > 10x - x$

33.  $2a - (-3)^2 \geq 13$

34.  $6 - \frac{x}{3} + 1 > \frac{2}{3}$

35.  $12(x-3) + 2x > 6$

36.  $15 \geq 19 + 2(q-18)$

37. **Communications** One cell phone company offers a plan that costs \$29.99 and includes unlimited night and weekend minutes. Another company offers a plan that costs \$19.99 and charges \$0.35 per minute during nights and weekends. For what numbers of night and weekend minutes does the second company's plan cost more than the first company's plan?

Solve each inequality and graph the solutions.

38.  $-12 > -4x - 8$

39.  $5x + 4 \leq 14$

40.  $\frac{2}{3}x - 5 > 7$

41.  $x - 3x > 2 - 10$

42.  $5 - x - 2 > 3$

43.  $3 < 2x - 5(x+3)$

44.  $\frac{1}{6} - \frac{2}{3}m \geq \frac{1}{4}$

45.  $4 - (r-2) > 3 - 5$

46.  $0.3 - 0.5n + 1 \geq 0.4$

47.  $6^2 > 4(x+2)$

48.  $-4 - 2n + 4n > 7 - 2^2$

49.  $\frac{1}{4}(p-10) \geq 6 - 4$

50. Use the inequality  $-4t - 8 \leq 12$  to fill in the missing numbers.

a.  $t \geq \blacksquare$

b.  $t + 4 \geq \blacksquare$

c.  $t - \blacksquare \geq 0$

d.  $t + 10 \geq \blacksquare$

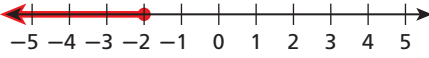
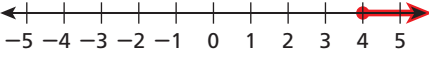

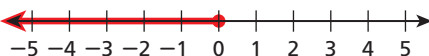
e.  $3t \geq \blacksquare$

f.  $\frac{t}{\blacksquare} \geq -5$

Write an inequality for each statement. Solve the inequality and graph the solutions.

51. One-half of a number, increased by 9, is less than 33.
52. Six is less than or equal to the sum of 4 and  $-2x$ .
53. The product of 4 and the sum of a number and 12 is at most 16.
54. The sum of half a number and two-thirds of the number is less than 14.

Solve each inequality and match the solution to the correct graph.

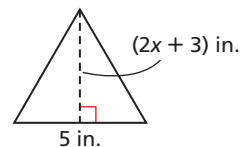
55.  $4x - 9 \geq 7$ 
  - A. 
56.  $-6 \geq 3(x - 2)$ 
  - B. 
57.  $-2x - 6 \geq -4 + 2$ 
  - C. 
58.  $\frac{1}{2} - \frac{1}{3}x \leq \left(\frac{2}{3} + \frac{1}{3}\right)^2$ 
  - D. 

59. **Entertainment** A digital video recorder (DVR) records television shows on an internal hard drive. To use a DVR, you need a subscription with a DVR service company. Two companies advertise their charges for a DVR machine and subscription service.



For what numbers of months will a consumer pay less for the machine and subscription at Easy Electronics than at Cable Solutions?

60. **Geometry** The area of the triangle shown is less than 55 square inches.
  - a. Write an inequality that can be used to find  $x$ .
  - b. Solve the inequality you wrote in part a.
  - c. What is the maximum height of the triangle?



### Real-World Connections



61. a. A band wants to create a CD of their last concert. They received a donation of \$500 to cover the cost. The total cost is \$350 plus \$3 per CD. Complete the table to find a relationship between the number of CDs and the total cost.
 

Number	Process	Cost
1	$350 + 3$	353
2	■	■
3	■	■
10	■	■
$n$	■	
- b. Write an equation for the cost  $c$  of the CDs based on the number of CDs  $n$ .
- c. Write an inequality that can be used to determine how many CDs can be made with the \$500 donation. Solve the inequality and determine how many CDs the band can have made from the \$500 donation.

**H.O.T.** 62. **Critical Thinking** What is the least whole number that is a solution of  $4r - 4.9 > 14.95$ ?

**H.O.T.** 63. **Write About It** Describe two sets of steps to solve  $2(x + 3) > 10$ .

## TEST PREP

64. What are the solutions of  $3y > 2x + 4$  when  $y = 6$ ?

- (A)  $7 > x$       (B)  $x > 7$       (C)  $x > 11$       (D)  $11 > x$

65. Cecilia has \$30 to spend at a carnival. Admission costs \$5.00, lunch will cost \$6.00, and each ride ticket costs \$1.25. Which inequality represents the number of ride tickets  $x$  that Cecilia can buy?

- (F)  $30 - (5 - 6) + 1.25x \leq 30$       (H)  $30 - (5 + 6) \leq 1.25x$   
 (G)  $5 + 6 + 1.25x \leq 30$       (J)  $30 + 1.25x \leq 5 + 6$

66. Which statement is modeled by  $2p + 5 < 11$ ?

- (A) The sum of 5 and 2 times  $p$  is at least 11.  
 (B) Five added to the product of 2 and  $p$  is less than 11.  
 (C) Two times  $p$  plus 5 is at most 11.  
 (D) The product of 2 and  $p$  added to 5 is 11.

67. **Gridded Response** A basketball team scored 8 points more in its second game than in its first. In its third game, the team scored 42 points. The total number of points scored in the three games was more than 150. What is the least number of points the team might have scored in its *second* game?

## CHALLENGE AND EXTEND

Solve each inequality and graph the solutions.

68.  $3(x + 2) - 6x + 6 \leq 0$       69.  $-18 > -(2x + 9) - 4 + x$       70.  $\frac{2 + x}{2} - (x - 1) > 1$

Write an inequality for each statement. Graph the solutions.

71.  $x$  is a positive number.      72.  $x$  is a negative number.  
 73.  $x$  is a nonnegative number.      74.  $x$  is not a positive number.  
 75.  $x$  times negative 3 is positive.      76. The opposite of  $x$  is greater than 2.

MATHEMATICAL PRACTICES

## FOCUS ON MATHEMATICAL PRACTICES

**H.O.T.** 77. **Modeling** Mario wants to spend no more than \$85 per month for texting. He is considering a plan that provides him with 200 free text messages for \$40 per month, plus \$0.10 for each additional text sent or received.

- a. Complete the table to show how much Mario would pay for each number of text messages sent or received.

Number of messages	200	400	600	800	1000
Cost in dollars	■	■	■	■	■

- b. Based on the table, write and solve an inequality that represents the maximum number of texts that Mario can send or receive under this plan.



# 5-2

## Solving Inequalities with Variables on Both Sides



**Essential Question:** How can you solve inequalities that have the variable on both sides?

### Objective

Solve inequalities that contain variable terms on both sides.

### Who uses this?

Business owners can use inequalities to find the most cost-efficient services. (See Example 2.)

Some inequalities have variable terms on both sides of the inequality symbol. You can solve these inequalities like you solved equations with variables on both sides.

Use the properties of inequality to “collect” all the variable terms on one side and all the constant terms on the other side.

**COMMON CORE GPS** **EXAMPLE**  
MCC9-12.A.REI.3

### 1 Solving Inequalities with Variables on Both Sides

Solve each inequality and graph the solutions.



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**A**  $x < 3x + 8$

$$x < 3x + 8$$

$$\begin{array}{r} -x \\ \hline 0 < 2x + 8 \end{array}$$

$$\begin{array}{r} -8 \\ \hline -8 < 2x \end{array}$$

$$\frac{-8}{2} < \frac{2x}{2}$$

$$-4 < x \text{ (or } x > -4)$$



To collect the variable terms on one side, subtract  $x$  from both sides.

Since 8 is added to  $2x$ , subtract 8 from both sides to undo the addition.

Since  $x$  is multiplied by 2, divide both sides by 2 to undo the multiplication.

**B**  $6x - 1 \leq 3.5x + 4$

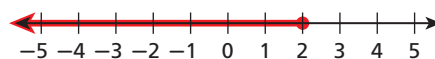
$$6x - 1 \leq 3.5x + 4$$

$$\begin{array}{r} -6x \\ \hline -1 \leq -2.5x + 4 \end{array}$$

$$\begin{array}{r} -4 \\ \hline -5 \leq -2.5x \end{array}$$

$$\frac{-5}{-2.5} \geq \frac{-2.5x}{-2.5}$$

$$2 \geq x$$



Subtract  $6x$  from both sides.

Since 4 is added to  $-2.5x$ , subtract 4 from both sides to undo the addition.

Since  $x$  is multiplied by  $-2.5$ , divide both sides by  $-2.5$  to undo the multiplication. Reverse the inequality symbol.

### Helpful Hint

When you divide by a negative number, remember to reverse the inequality symbol.



Solve each inequality and graph the solutions.

1a.  $4x \geq 7x + 6$

1b.  $5t + 1 < -2t - 6$

## Business Application

The *Daily Info* charges a fee of \$650 plus \$80 per week to run an ad. The *People's Paper* charges \$145 per week. For how many weeks will the total cost at *Daily Info* be less expensive than the cost at *People's Paper*?



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Let  $w$  be the number of weeks the ad runs in the paper.

<i>Daily Info</i> fee	plus	\$80 per week	times	number of weeks	is less expensive than	<i>People's Paper</i> charge per week	times	number of weeks.
\$650	+	\$80	•	$w$	<	\$145	•	$w$

$$650 + 80w < 145w$$

$$\begin{array}{r} \phantom{650} \\ \underline{-80w} \phantom{0} \\ 650 \phantom{0} \end{array} < \begin{array}{r} \phantom{650} \\ \underline{-80w} \\ 65w \end{array}$$

$$650 < 65w$$

$$\frac{650}{65} < \frac{65w}{65}$$

$$10 < w$$

Subtract  $80w$  from both sides.

Since  $w$  is multiplied by 65, divide both sides by 65 to undo the multiplication.

The total cost at *Daily Info* is less than the cost at *People's Paper* if the ad runs for more than 10 weeks.



2. A-Plus Advertising charges a fee of \$24 plus \$0.10 per flyer to print and deliver flyers. Print and More charges \$0.25 per flyer. For how many flyers is the cost at A-Plus Advertising less than the cost at Print and More?

You may need to simplify one or both sides of an inequality before solving it. Look for like terms to combine and places to use Distributive Property.

## Simplifying Each Side Before Solving

Solve each inequality and graph the solutions.

**A**  $6(1 - x) < 3x$

$$6(1 - x) < 3x$$

$$6(1) - 6(x) < 3x$$

$$6 - 6x < 3x$$

$$\begin{array}{r} \phantom{6} \\ \underline{+6x} \phantom{0} \\ 6 \phantom{0} \end{array} < \begin{array}{r} \phantom{6} \\ \underline{+6x} \\ 9x \end{array}$$

$$6 < 9x$$

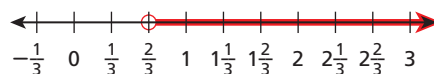
$$\frac{6}{9} < \frac{9x}{9}$$

$$\frac{2}{3} < x$$

Distribute 6 on the left side of the inequality.

Add  $6x$  to both sides so that the coefficient of  $x$  is positive.

Since  $x$  is multiplied by 9, divide both sides by 9 to undo the multiplication.



## Helpful Hint

In Example 3B, you can also multiply each term in the inequality by the same power of 10 to clear the decimals.

$$\begin{array}{r} 10(1.6x) \leq 10(-0.2x) \\ \quad \quad \quad + 10(0.9) \\ 16x \leq -2x + 9 \end{array}$$

Solve each inequality and graph the solutions.

**B**  $1.6x \leq -0.2x + 0.9$

$$\begin{array}{r} 1.6x \leq -0.2x + 0.9 \\ + 0.2x \quad + 0.2x \\ \hline 1.8x \leq \quad \quad 0.9 \end{array}$$

$$\frac{1.8x}{1.8} \leq \frac{0.9}{1.8}$$

$$x \leq \frac{1}{2}$$

Since  $-0.2x$  is added to  $0.9$ , subtract  $-0.2x$  from both sides. Subtracting  $-0.2x$  is the same as adding  $0.2x$ .

Since  $x$  is multiplied by  $1.8$ , divide both sides by  $1.8$  to undo the multiplication.



Solve each inequality and graph the solutions. Check your answer.

**3a.**  $5(2 - r) \geq 3(r - 2)$

**3b.**  $0.5x - 0.3 + 1.9x < 0.3x + 6$

Some inequalities are true no matter what value is substituted for the variable. For these inequalities, all real numbers are solutions.

Some inequalities are false no matter what value is substituted for the variable. These inequalities have no solutions.

If both sides of an inequality are fully simplified and the same variable term appears on both sides, then the inequality has all real numbers as solutions or it has no solutions. Look at the other terms in the inequality to decide which is the case.

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## EXAMPLE

MCC9-12.A.REI.3

4

### All Real Numbers as Solutions or No Solutions

Solve each inequality.

**A**  $x + 5 \geq x + 3$

$$x + 5 \geq x + 3$$

The same variable term ( $x$ ) appears on both sides. Look at the other terms.

For any number  $x$ , adding  $5$  will always result in a greater number than adding  $3$ .

All values of  $x$  make the inequality true.  
All real numbers are solutions.

**B**  $2(x + 3) < 5 + 2x$

$$2x + 6 < 5 + 2x \quad \text{Distribute 2 on the left side.}$$

The same variable term ( $2x$ ) appears on both sides. Look at the other terms.

For any number  $2x$ , adding  $6$  will never result in a lesser number than adding  $5$ .

No values of  $x$  make the inequality true.  
There are no solutions.

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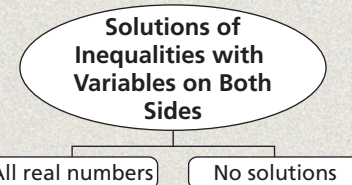
Solve each inequality.

**4a.**  $4(y - 1) \geq 4y + 2$

**4b.**  $x - 2 < x + 1$

## THINK AND DISCUSS

1. Explain how you would collect the variable terms to solve the inequality  $5c - 4 > 8c + 2$ .
2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, give an example of an inequality of the indicated type.



## 5-2

## Exercises



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## GUIDED PRACTICE

SEE EXAMPLE 1 Solve each inequality and graph the solutions.

- |                          |                                 |
|--------------------------|---------------------------------|
| 1. $2x > 4x - 6$         | 2. $7y + 1 \leq y - 5$          |
| 3. $27x + 33 > 58x - 29$ | 4. $-3r < 10 - r$               |
| 5. $5c - 4 > 8c + 2$     | 6. $4.5x - 3.8 \geq 1.5x - 2.3$ |

SEE EXAMPLE 2 7. **School** The school band will sell pizzas to raise money for new uniforms. The supplier charges \$100 plus \$4 per pizza. If the band members sell the pizzas for \$7 each, how many pizzas will they have to sell to make a profit?

SEE EXAMPLE 3 Solve each inequality and graph the solutions.

- |  |                              |
|--|------------------------------|
| 8. $5(4 + x) \leq 3(2 + x)$                        | 9. $-4(3 - p) > 5(p + 1)$    |
| 10. $2(6 - x) < 4x$                                | 11. $4x > 3(7 - x)$          |
| 12. $\frac{1}{2}f + \frac{3}{4} \geq \frac{1}{4}f$ | 13. $-36.72 + 5.65t < 0.25t$ |

SEE EXAMPLE 4 Solve each inequality.

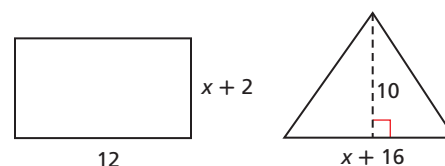
- |                               |                         |                             |
|-------------------------------|-------------------------|-----------------------------|
| 14. $2(x - 2) \leq -2(1 - x)$ | 15. $4(y + 1) < 4y + 2$ | 16. $4v + 1 < 4v - 7$       |
| 17. $b - 4 \geq b - 6$        | 18. $3(x - 5) > 3x$     | 19. $2k + 7 \geq 2(k + 14)$ |

## PRACTICE AND PROBLEM SOLVING

Solve each inequality and graph the solutions.

- |                      |                       |                               |
|----------------------|-----------------------|-------------------------------|
| 20. $3x \leq 5x + 8$ | 21. $9y + 3 > 4y - 7$ | 22. $1.5x - 1.2 < 3.1x - 2.8$ |
| 23. $7 + 4b \geq 3b$ | 24. $7 - 5t < 4t - 2$ | 25. $2.8m - 5.2 > 0.8m + 4.8$ |

26. **Geometry** For what values of  $x$  is the area of the rectangle greater than the area of the triangle?



**Independent Practice**

For Exercises	See Example
20–25	1
26	2
27–32	3
33–38	4

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Online Extra Practice

Solve each inequality and graph the solutions.

27.  $4(2 - x) \leq 5(x - 2)$       28.  $-3(n + 4) < 6(1 - n)$       29.  $9(w + 2) \leq 12w$   
 30.  $4.5 + 1.3t > 3.8t - 3$       31.  $\frac{1}{2}r + \frac{2}{3} \geq \frac{1}{3}r$       32.  $2(4 - n) < 3n - 7$

Solve each inequality.

33.  $3(2 - x) < -3(x - 1)$       34.  $7 - y > 5 - y$       35.  $3(10 + z) \leq 3z + 36$   
 36.  $-5(k - 1) \geq 5(2 - k)$       37.  $4(x - 1) \leq 4x$       38.  $3(v - 9) \geq 15 + 3v$

Solve each inequality and graph the solutions.

39.  $3t - 12 > 5t + 2$       40.  $-5(y + 3) - 6 < y + 3$   
 41.  $3x + 9 - 5x < x$       42.  $18 + 9p > 12p - 31$   
 43.  $2(x - 5) < -3x$       44.  $-\frac{2}{5}x \leq \frac{4}{5} - \frac{3}{5}x$   
 45.  $-2(x - 7) - 4 - x < 8x + 32$       46.  $-3(2r - 4) \geq 2(5 - 3r)$   
 47.  $-7x - 10 + 5x \geq 3(x + 4) + 8$       48.  $-\frac{1}{3}(n + 8) + \frac{1}{3}n \leq 1 - n$



**Recreation**

49. **Recreation** A red kite is 100 feet off the ground and is rising at 8 feet per second. A blue kite is 180 feet off the ground and is rising at 5 feet per second. How long will it take for the red kite to be higher than the blue kite? Round your answer to the nearest second.

50. **Education** The table shows the enrollment in Howard High School and Phillips High School for three school years.

School Enrollment			
	Year 1	Year 2	Year 3
Howard High School	1192	1188	1184
Phillips High School	921	941	961

- How much did the enrollment change each year at Howard?
- Use the enrollment in year 1 and your answer from part a to write an expression for the enrollment at Howard in any year  $x$ .
- How much did the enrollment change each year at Phillips?
- Use the enrollment in year 1 and your answer from part c to write an expression for the enrollment at Phillips in any year  $x$ .
- Assume that the pattern in the table continues. Use your expressions from parts b and d to write an inequality that can be solved to find the year in which the enrollment at Phillips High School will be greater than the enrollment at Howard High School. Solve your inequality and graph the solutions.

The American Kitefliers Association has over 4000 members in 35 countries. Kitefliers participate in festivals, competitions, and kite-making workshops.



**Real-World Connections**

51. **a.** The school orchestra is creating a CD of their last concert. The total cost is  $\$400 + 4.50$  per CD. Write an expression for the cost of creating the CDs based on the number of CDs  $n$ .  
**b.** The orchestra plans to sell the CDs for  $\$12$ . Write an expression for the amount the orchestra earns from the sale of  $n$  CDs.  
**c.** In order for the orchestra to make a profit, the amount they make selling the CDs must be greater than the cost of creating the CDs. Write an inequality that can be solved to find the number of CDs the orchestra must sell in order to make a profit. Solve your inequality.



Write an inequality to represent each relationship. Solve your inequality.

52. Four more than twice a number is greater than two-thirds of the number.
53. Ten less than five times a number is less than six times the number decreased by eight.
54. The sum of a number and twenty is less than four times the number decreased by one.
55. Three-fourths of a number is greater than or equal to five less than the number.
56. **Entertainment** Use the table to determine how many movies you would have to rent for Video View to be less expensive than Movie Place.

	Membership Fee (\$)	Cost per Rental (\$)
Movie Place	None	2.99
Video View	19.99	1.99

57. **Geometry** In an acute triangle, all angles measure less than  $90^\circ$ . Also, the sum of the measures of any two angles is greater than the measure of the third angle. Can the measures of an acute triangle be  $x$ ,  $x - 1$ , and  $2x$ ? Explain.
- HOT.** 58. **Write About It** Compare the steps you would follow to solve an inequality to the steps you would follow to solve an equation.
- HOT.** 59. **Critical Thinking** How can you tell just by looking at the inequality  $x > x + 1$  that it has no solutions?
- HOT.** 60. **/// ERROR ANALYSIS ///** Two students solved the inequality  $5x < 3 - 4x$ . Which is incorrect? Explain the error.


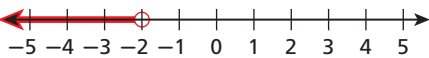

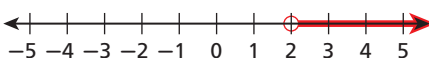
**A**

$$\begin{array}{r} 5x < 3 - 4x \\ + 4x \quad + 4x \\ \hline 9x < 3 \\ x < \frac{1}{3} \end{array}$$

**B**

$$\begin{array}{r} 5x < 3 - 4x \\ - 4x \quad - 4x \\ \hline x < 3 \end{array}$$

## TEST PREP

61. If  $a - b > a + b$ , which statement is true?
- (A) The value of  $a$  is positive.      (C) The value of  $a$  is negative.  
 (B) The value of  $b$  is positive.      (D) The value of  $b$  is negative.
62. If  $-a < b$ , which statement is always true?
- (F)  $a < b$       (G)  $a > b$       (H)  $a < -b$       (J)  $a > -b$
63. Which is a solution of the inequality  $7(2 - x) > 4(x - 2)$ ?
- (A)  $-2$       (B)  $2$       (C)  $4$       (D)  $7$
64. Which is the graph of  $-5x < -2x - 6$ ?
- (F)  (H) 
- (G)  (J) 

65. **Short Response** Write a real-world situation that could be modeled by the inequality  $7x + 4 > 4x + 13$ . Explain how the inequality relates to your situation.

## CHALLENGE AND EXTEND

Solve each inequality.

66.  $2\frac{1}{2} + 2x \geq 5\frac{1}{2} + 2\frac{1}{2}x$
67.  $1.6x - 20.7 > 6.3x - (-2.2x)$
68.  $1.3x - 7.5x < 8.5x - 29.4$
69.  $-4w + \frac{-8 - 37}{9} \leq \frac{75 - 3}{9} + 3w$
70. Replace the square and circle with numbers so that the inequality has all real numbers as solutions.  $\square - 2x < \bigcirc - 2x$
71. Replace the square and circle with numbers so that the inequality has no solutions.  $\square - 2x < \bigcirc - 2x$
- H.O.T.** 72. **Critical Thinking** Explain whether there are any numbers that can replace the square and circle so that the inequality has all real numbers as solutions.  $\square + 2x < \bigcirc + x$

### MATHEMATICAL PRACTICES

## FOCUS ON MATHEMATICAL PRACTICES

- H.O.T.** 73. **Analysis** The table below shows a step-by-step solution to the inequality  $2x + 5 > 7x - 35$ . Fill in the remaining inequality symbols and steps.

Left Side	Symbol	Right Side	Step
$2x + 5$	$>$	$7x - 35$	None
$2x$		$7x - 40$	Subtract 5
$-5x$		$-40$	
$x$		$8$	

## Career Path



**Katie Flannigan**  
Culinary Arts program

**Q:** What math classes did you take in high school?

**A:** Algebra 1, Geometry, and Algebra 2

**Q:** What math classes have you taken since high school?

**A:** I have taken a basic accounting class and a business math class.

**Q:** How do you use math?

**A:** I use math to estimate how much food I need to buy. I also use math when adjusting recipe amounts to feed large groups of people.

**Q:** What are your future plans?

**A:** I plan to start my own catering business. The math classes I took will help me manage the financial aspects of my business.

# 5-3

## Solving Compound Inequalities

**Essential Question:** How can you solve compound inequalities and graph their solutions?

### Objectives

Solve compound inequalities in one variable.

Graph solution sets of compound inequalities in one variable.

### Vocabulary

compound inequality  
intersection  
union

### Who uses this?

A lifeguard can use compound inequalities to describe the safe pH levels in a swimming pool. (See Example 1.)

The inequalities you have seen so far are simple inequalities. When two simple inequalities are combined into one statement by the words AND or OR, the result is called a **compound inequality**.



### Compound Inequalities

WORDS	ALGEBRA	GRAPH
All real numbers greater than 2 AND less than 6	$x > 2$ AND $x < 6$ $2 < x < 6$	
All real numbers greater than or equal to 2 AND less than or equal to 6	$x \geq 2$ AND $x \leq 6$ $2 \leq x \leq 6$	
All real numbers less than 2 OR greater than 6	$x < 2$ OR $x > 6$	
All real numbers less than or equal to 2 OR greater than or equal to 6	$x \leq 2$ OR $x \geq 6$	

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MCC9-12.A.CED.1

### EXAMPLE

### 1 Chemistry Application

A water analyst recommends that the pH level of swimming pool water be between 7.2 and 7.6 inclusive. Write a compound inequality to show the pH levels that are within the recommended range. Graph the solutions.

Let  $p$  be the pH level of swimming pool water.



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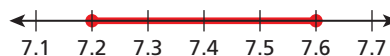
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### Helpful Hint

The phrase "between 7.2 and 7.6 inclusive" means 7.2 and 7.6 are solutions. Use a solid circle for endpoints that are solutions.

7.2 is less than or equal to pH level is less than or equal to 7.6

$7.2 \leq p \leq 7.6$

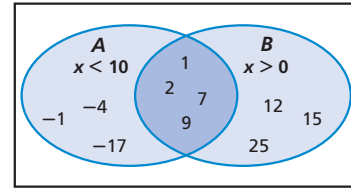




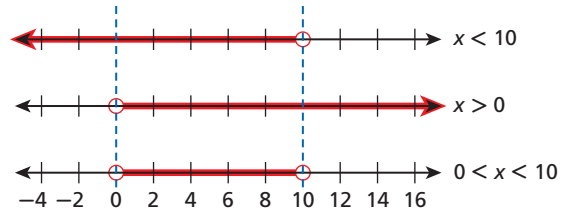


1. The free chlorine level in a pool should be between 1.0 and 3.0 parts per million inclusive. Write a compound inequality to show the levels that are within this range. Graph the solutions.

In this diagram, oval A represents some integer solutions of  $x < 10$ , and oval B represents some integer solutions of  $x > 0$ . The overlapping region represents numbers that belong in both ovals. Those numbers are solutions of *both*  $x < 10$  and  $x > 0$ .



You can graph the solutions of a compound inequality involving AND by using the idea of an overlapping region. The overlapping region is called the **intersection** and shows the numbers that are solutions of both inequalities.



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## EXAMPLE

MCC9-12.A.REI.3

2

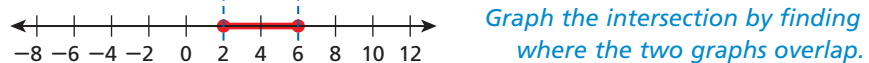
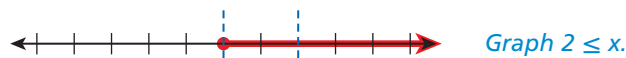
### Solving Compound Inequalities Involving AND

Solve each compound inequality and graph the solutions.

**A**  $4 \leq x + 2 \leq 8$

$4 \leq x + 2$  AND  $x + 2 \leq 8$  Write the compound inequality using AND.

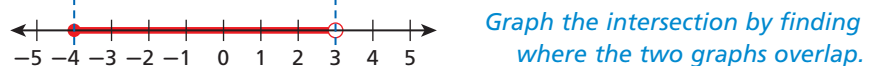
$\frac{-2}{-2} \frac{-2}{-2} \frac{-2}{-2} \frac{-2}{-2}$  Solve each simple inequality.  
 $2 \leq x$  AND  $x \leq 6$



**B**  $-5 \leq 2x + 3 < 9$

$-5 \leq 2x + 3 < 9$  Since 3 is added to  $2x$ , subtract 3 from each part of the inequality.

$\frac{-3}{-3} \frac{-3}{-3} \frac{-3}{-3}$   
 $-8 \leq 2x < 6$   
 $\frac{-8}{2} \leq \frac{2x}{2} < \frac{6}{2}$  Since  $x$  is multiplied by 2, divide each part of the inequality by 2.  
 $-4 \leq x < 3$



### Remember!

The statement  $-5 \leq 2x + 3 \leq 9$  consists of two inequalities connected by AND. Example 2B shows a "shorthand" method for solving this type of inequality.

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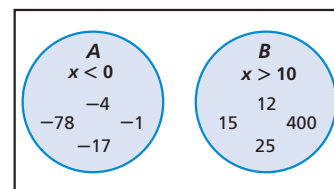


Solve each compound inequality and graph the solutions.

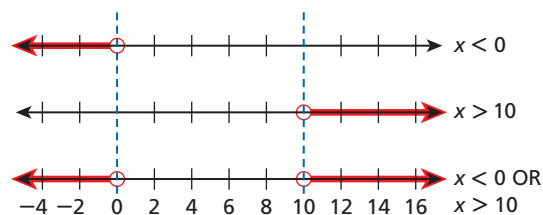
2a.  $-9 < x - 10 < -5$

2b.  $-4 \leq 3n + 5 < 11$

In this diagram, circle  $A$  represents some integer solutions of  $x < 0$ , and circle  $B$  represents some integer solutions of  $x > 10$ . The combined shaded regions represent numbers that are solutions of *either*  $x < 0$  or  $x > 10$ .



You can graph the solutions of a compound inequality involving OR by using the idea of combining regions. The combined regions are called the **union** and show the numbers that are solutions of either inequality.



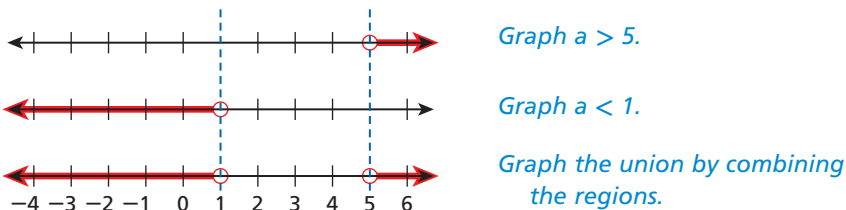
**COMMON CORE GPS** **EXAMPLE** 3 **Solving Compound Inequalities Involving OR**  
MCC9-12.A.REI.3

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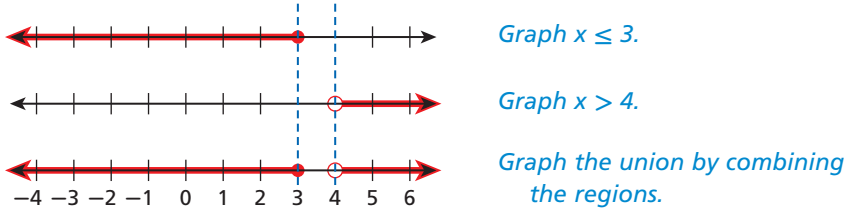
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Solve each compound inequality and graph the solutions.

**A**  $-4 + a > 1$  OR  $-4 + a < -3$   
 $-4 + a > 1$  OR  $-4 + a < -3$   
 $\frac{-4 + a}{+4} \frac{+4}{+4} \frac{+4}{+4} \frac{+4}{+4}$  *Solve each simple inequality.*  
 $a > 5$  OR  $a < 1$



**B**  $2x \leq 6$  OR  $3x > 12$   
 $2x \leq 6$  OR  $3x > 12$   
 $\frac{2x}{2} \leq \frac{6}{2}$   $\frac{3x}{3} > \frac{12}{3}$  *Solve each simple inequality.*  
 $x \leq 3$  OR  $x > 4$



**CHECK IT OUT!** Solve each compound inequality and graph the solutions.

- 3a.**  $2 + r < 12$  OR  $r + 5 > 19$   
**3b.**  $7x \geq 21$  OR  $2x < -2$

Every solution of a compound inequality involving AND must be a solution of both parts of the compound inequality. If no numbers are solutions of *both* simple inequalities, then the compound inequality has no solutions.

The solutions of a compound inequality involving OR are not always two separate sets of numbers. There may be numbers that are solutions of both parts of the compound inequality.

## Writing a Compound Inequality from a Graph

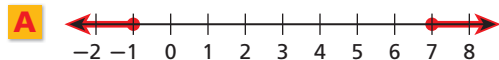


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Write the compound inequality shown by each graph.



The shaded portion of the graph is not between two values, so the compound inequality involves OR.

On the left, the graph shows an arrow pointing left, so use either  $<$  or  $\leq$ .  
The solid circle at  $-1$  means  $-1$  is a solution, so use  $\leq$ .

$$x \leq -1$$

On the right, the graph shows an arrow pointing right, so use either  $>$  or  $\geq$ .  
The solid circle at  $7$  means  $7$  is a solution, so use  $\geq$ .

$$x \geq 7$$

The compound inequality is  $x \leq -1$  OR  $x \geq 7$ .



The shaded portion of the graph is between the values 0 and 6, so the compound inequality involves AND.

The shaded values are to the right of 0, so use  $>$  or  $\geq$ .  
The solid circle at 0 means 0 is a solution, so use  $\geq$ .

$$x \geq 0$$

The shaded values are to the left of 6, so use  $<$  or  $\leq$ .  
The empty circle at 6 means 6 is not a solution, so use  $<$ .

$$x < 6$$

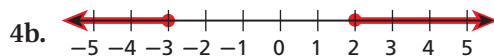
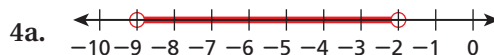
The compound inequality is  $x \geq 0$  AND  $x < 6$ .

## Writing Math

The compound inequality in Example 4B can also be written with the variable between the two endpoints.  
 $0 \leq x < 6$



Write the compound inequality shown by the graph.

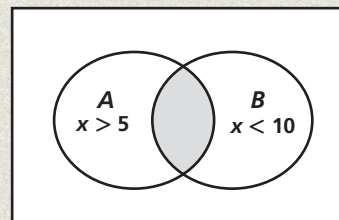


MCC.MP.2

MATHEMATICAL PRACTICES

## THINK AND DISCUSS

- Describe how to write the compound inequality  $y > 4$  AND  $y \leq 12$  without using the joining word AND.
- GET ORGANIZED** Copy and complete the graphic organizers. Write three solutions in each of the three sections of the diagram. Then write each of your nine solutions in the appropriate column or columns of the table.



$x > 5$ AND $x < 10$	$x > 5$ OR $x < 10$



## GUIDED PRACTICE

1. **Vocabulary** The graph of a(n) \_\_\_?\_\_\_ shows all values that are solutions to both simple inequalities that make a compound inequality. (*union* or *intersection*)
2. **Biology** An iguana needs to live in a warm environment. The temperature in a pet iguana's cage should be between 70 °F and 95 °F inclusive. Write a compound inequality to show the temperatures that are within the recommended range. Graph the solutions.

SEE EXAMPLE 1

Solve each compound inequality and graph the solutions.

SEE EXAMPLE 2

3.  $-3 < x + 2 < 7$

4.  $5 \leq 4x + 1 \leq 13$

5.  $2 < x + 2 < 5$

6.  $11 < 2x + 3 < 21$

SEE EXAMPLE 3

7.  $x + 2 < -6$  OR  $x + 2 > 6$

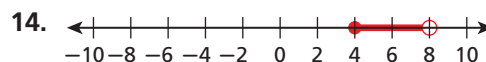
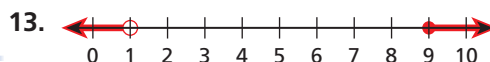
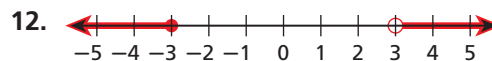
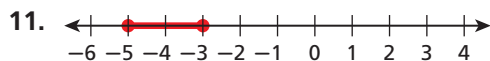
8.  $r - 1 < 0$  OR  $r - 1 > 4$

9.  $n + 2 < 3$  OR  $n + 3 > 7$

10.  $x - 1 < -1$  OR  $x - 5 > -1$

SEE EXAMPLE 4

Write the compound inequality shown by each graph.



## PRACTICE AND PROBLEM SOLVING

## Independent Practice

For Exercises	See Example
15	1
16–19	2
20–23	3
24–27	4



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Online Extra Practice

15. **Meteorology** One layer of Earth's atmosphere is called the stratosphere. At one point above Earth's surface the stratosphere extends from an altitude of 16 km to an altitude of 50 km. Write a compound inequality to show the altitudes that are within the range of the stratosphere. Graph the solutions.

Solve each compound inequality and graph the solutions.

16.  $-1 < x + 1 < 1$

17.  $1 \leq 2n - 5 \leq 7$

18.  $-2 < x - 2 < 2$

19.  $5 < 3x - 1 < 17$

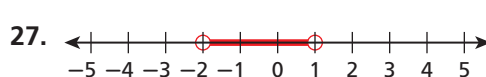
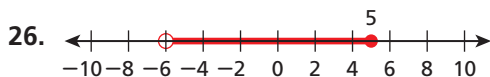
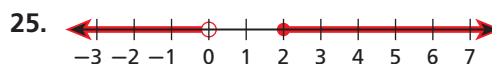
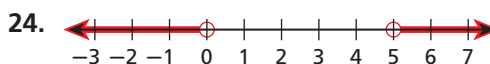
20.  $x - 4 < -7$  OR  $x + 3 > 4$

21.  $2x + 1 < 1$  OR  $x + 5 > 8$

22.  $x + 1 < 2$  OR  $x + 5 > 8$

23.  $x + 3 < 0$  OR  $x - 2 > 0$

Write the compound inequality shown by each graph.



28. **Music** A typical acoustic guitar has a range of three octaves. When the guitar is tuned to "concert pitch," the range of frequencies for those three octaves is between 82.4 Hz and 659.2 Hz inclusive. Write a compound inequality to show the frequencies that are within the range of a typical acoustic guitar. Graph the solutions.

## Real-World Connections



29. Jenna's band is going to record a CD at a recording studio. They will pay \$225 to use the studio for one day and \$80 per hour for sound technicians. Jenna has \$200 and can reasonably expect to raise up to an additional \$350 by taking pre-orders for the CDs.
- Explain how the inequality  $200 \leq 225 + 80n \leq 550$  can be used to find the number of hours Jenna and her band can afford to use the studio and sound technicians.
  - Solve the inequality. Are there any numbers in the solution set that are not reasonable in this situation?
  - Suppose Jenna raises \$350 in pre-orders. How much more money would she need to raise if she wanted to use the studio and sound technicians for 6 hours?

Write and graph a compound inequality for the numbers described.

- all real numbers between  $-6$  and  $6$
- all real numbers less than or equal to  $2$  and greater than or equal to  $1$
- all real numbers greater than  $0$  and less than  $15$
- all real numbers between  $-10$  and  $10$  inclusive



## Chemistry



The element gallium is in a solid state at room temperature but becomes a liquid at about  $30^\circ\text{C}$ . Gallium stays in a liquid state until it reaches a temperature of about  $2204^\circ\text{C}$ .

34. **Transportation** The cruise-control function on Georgina's car should keep the speed of the car within  $3$  mi/h of the set speed. Write a compound inequality to show the acceptable speeds  $s$  if the set speed is  $55$  mi/h. Graph the solutions.
35. **Chemistry** Water is not a liquid if its temperature is above  $100^\circ\text{C}$  or below  $0^\circ\text{C}$ . Write a compound inequality for the temperatures  $t$  when water is not a liquid.

Solve each compound inequality and graph the solutions.

- $5 \leq 4b - 3 \leq 9$
- $r + 2 < -2$  OR  $r - 2 > 2$
- $x - 4 \geq 5$  AND  $x - 4 \leq 5$
- $-3 < x - 1 < 4$
- $2a - 5 < -5$  OR  $3a - 2 > 1$
- $n - 4 < -2$  OR  $n + 1 > 6$
- Sports** The ball used in a soccer game may not weigh more than  $16$  ounces or less than  $14$  ounces at the start of the match. After  $1\frac{1}{2}$  ounces of air was added to a ball, the ball was approved for use in a game. Write and solve a compound inequality to show how much the ball might have weighed before the air was added.

43. **Meteorology** Tornado damage is rated using the Fujita scale shown in the table. A tornado has a wind speed of  $200$  miles per hour. Write and solve a compound inequality to show how many miles per hour the wind speed would need to increase for the tornado to be rated "devastating" but not "incredible."

Fujita Tornado Scale		
Category	Type	Wind Speed (mi/h)
F0	Weak	40 to 72
F1	Moderate	73 to 112
F2	Significant	113 to 157
F3	Severe	158 to 206
F4	Devastating	207 to 260
F5	Incredible	261 to 318

44. Give a real-world situation that can be described by a compound inequality. Write the inequality that describes your situation.

- H.O.T.** 45. **Write About It** How are the graphs of the compound inequality  $x < 3$  AND  $x < 7$  and the compound inequality  $x < 3$  OR  $x < 7$  different? How are the graphs alike? Explain.

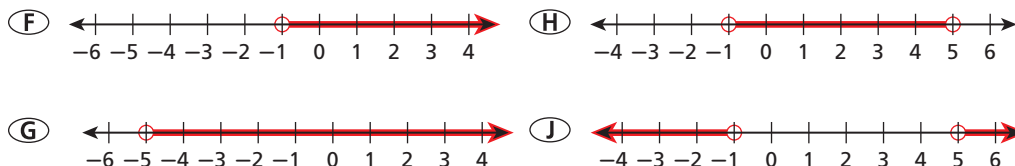
- H.O.T.** 46. **Critical Thinking** If there is no solution to a compound inequality, does the compound inequality involve OR or AND? Explain.

## TEST PREP

47. Which of the following describes the solutions of  $-x + 1 > 2$  OR  $x - 1 > 2$ ?

- (A) all real numbers greater than 1 or less than 3  
 (B) all real numbers greater than 3 or less than 1  
 (C) all real numbers greater than  $-1$  or less than 3  
 (D) all real numbers greater than 3 or less than  $-1$

48. Which of the following is a graph of the solutions of  $x - 3 < 2$  AND  $x + 3 > 2$ ?



49. Which compound inequality is shown by the graph?



- (A)  $x \leq 2$  OR  $x > 5$  (C)  $x \leq 2$  OR  $x \geq 5$   
 (B)  $x < 2$  OR  $x \geq 5$  (D)  $x \geq 2$  OR  $x > 5$
50. Which of the following is a solution of  $x + 1 \geq 3$  AND  $x + 1 \leq 3$ ?
- (F) 0 (G) 1 (H) 2 (J) 3

## CHALLENGE AND EXTEND

Solve and graph each compound inequality.

51.  $2c - 10 < 5 - 3c < 7c$  52.  $5p - 10 < p + 6 < 3p$   
 53.  $2s \leq 18 - s$  OR  $5s \geq s + 36$  54.  $9 - x \geq 5x$  OR  $20 - 3x \leq 17$
55. Write a compound inequality that represents all values of  $x$  that are NOT solutions to  $x < -1$  OR  $x > 3$ .
56. For the compound inequality  $x + 2 \geq a$  AND  $x - 7 \leq b$ , find values of  $a$  and  $b$  for which the only solution is  $x = 1$ .

MATHEMATICAL PRACTICES

## FOCUS ON MATHEMATICAL PRACTICES

- H.O.T.** 57. **Modeling** Ronaldo purchased a gym membership at a special rate that allows him at most 15 workouts per month. He has a trainer who requires him to work out at least 9 days per month. In the first half of April, Ronaldo completed  $d_1$  workouts, with  $d_1 \leq 9$ . Using the variable  $d_2$ , write a compound inequality to describe how many times Ronaldo should work out in the second half of April.
- H.O.T.** 58. **Counterexample** While working on a problem involving inequalities, Loretta noticed  $12 \leq x \leq 16$  has 4 integer solutions, 12, 13, 14, and 15, but  $12 < x < 16$  has only 2 integer solutions, 13 and 14. She proposed that  $a \leq x \leq b$  always has 2 more integer solutions than  $a < x < b$  whenever  $a < b$ . Can you think of a counterexample to disprove Loretta's conjecture?

# Ready to Go On?



## 5-1 Solving Two-Step and Multi-Step Inequalities

Solve each inequality and graph the solutions.

1.  $2x + 3 < 9$

2.  $3t - 2 > 10$

3.  $7 \geq 1 - 6r$

Solve each inequality.

4.  $2(x - 3) > -1$

5.  $\frac{1}{3}a + \frac{1}{2} > \frac{2}{3}$

6.  $15 < 5(m - 7)$

7.  $2 + (-6) > 0.8p$

8. The average of Mindy's two test scores must be at least 92 to make an A in the class. Mindy got an 88 on her first test. What scores can she get on her second test to make an A in the class?
9. Carl's Cable Company charges \$55 for monthly service plus \$4 for each pay-per-view movie. Teleview Cable Company charges \$110 per month with no fee for movies. For what number of movies is the cost of Carl's Cable Company less than the cost of Teleview?

## 5-2 Solving Inequalities with Variables on Both Sides

Solve each inequality and graph the solutions.

10.  $5x < 3x + 8$

11.  $6p - 3 > 9p$

12.  $r - 8 \geq 3r - 12$

Solve each inequality.

13.  $3(y + 6) > 2(y + 4)$

14.  $4(5 - g) \geq g$

15.  $4x < 4(x - 1)$

16.  $3(1 - x) \geq -3(x + 2)$

17. Phillip has \$100 in the bank and deposits \$18 per month. Gil has \$145 in the bank and deposits \$15 per month. For how many months will Gil have a larger bank balance than Phillip?
18. Hanna has a savings account with a balance of \$210 and deposits \$16 per month. Faith has a savings account with a balance of \$175 and deposits \$20 per month. Write and solve an inequality to determine the number of months Hanna's account balance will be greater than Faith's account balance.

## 5-3 Solving Compound Inequalities

Solve each compound inequality and graph the solutions.

19.  $-2 \leq x + 3 < 9$

20.  $m + 2 < -1$  OR  $m - 2 > 6$

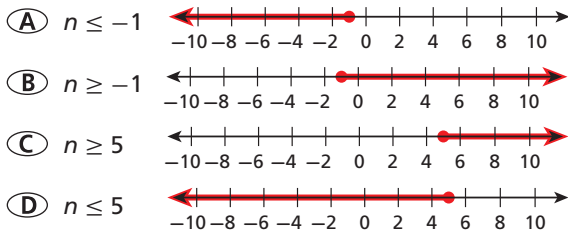
21.  $-3 \geq x - 1 > 2$

22.  $-2 > r + 2$  OR  $r + 4 < 5$

23. It is recommended that a certain medicine be stored in temperatures above 32 °F and below 70 °F. Write a compound inequality to show the acceptable storage temperatures for this medicine.

## Selected Response

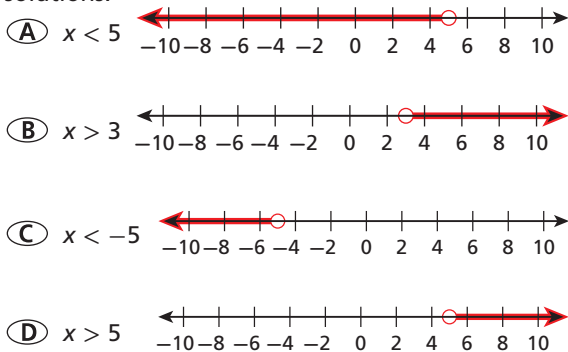
1. Solve the inequality  $3n - 6 - n \leq 4$  and graph the solutions.



2. A family travels to Bryce Canyon for three days. On the first day, they drove 150 miles. On the second day, they drove 190 miles. What is the least number of miles they drove on the third day if their average number of miles per day was at least 185?

- (F) 200 mi                      (H) 555 mi  
(G) 175 mi                      (J) 215 mi

3. Solve the inequality  $6x < 3x + 15$  and graph the solutions.



4. Mrs. Williams is deciding between two field trips for her class. The Science Center charges \$360 plus \$5 per student. The Dino Discovery Museum simply charges \$11 per student. For how many students will the Science Center charge less than the Dino Discovery Museum?

- (F) Fewer than 60 students  
(G) 354 or more students  
(H) More than 60 students  
(J) 354 or fewer students

5. Solve the inequality  $3(y - 3) \leq 3y + 2$ .

- (A)  $y \leq -1\frac{1}{6}$                       (C)  $y \leq 1\frac{5}{6}$   
(B) no solutions                      (D) All real numbers are solutions.

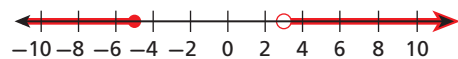
6. Fly with Us owns an airplane that has seats for 240 people. The company flies this airplane only if there are at least 100 people on the plane. Write a compound inequality to show the possible number of people in a flight on this airplane. Let  $n$  represent the possible number of people in the flight.

- (F)  $100 \geq n \geq 240$   
(G)  $100 \leq n \leq 240$   
(H)  $n \leq 240$   
(J)  $100 < n < 240$

7. Solve the compound inequality  $1 < 3x - 2 \leq 10$  and graph the solutions.

- (A)  $1 < x$  AND  $x \leq 4$                       (C)  $1 \leq x$  AND  $x \leq 4$   
(B)  $1 < x$  AND  $x < 4$                       (D)  $1 > x$  AND  $x \geq 4$

8. Write the compound inequality shown by the graph.



- (F)  $x < -5$  OR  $x > 3$                       (H)  $x \leq -5$  OR  $x > 3$   
(G)  $x \leq 3$  AND  $x > -5$                       (J)  $x \leq -5$  AND  $x > 3$

9. Which of the following is a solution of  $x - 9 < 5$  AND  $x + 5 \geq -1$ ?

- (A) 13                                      (C) 14  
(B) 16                                      (D) -7

## Mini-Tasks

10. A volleyball team scored 6 more points in its first game than in its third game. In the second game, the team scored 23 points. The total number of points scored was less than 55.

- a. Write and solve an inequality to find the number of points the team could have scored in its first game.
- b. Janie scored 8 points in the first game. Is it possible that she scored exactly half the team's points in that game? Explain.