UNIT 1



Choosing Appropriate Units



MCC9-12.N.Q.1 MCC9-12.A.CED.1, MCC9-12.N.Q.2 MCC9-12.N.Q.3

Contents

2-1	Rates, Ratios, and Proportions	32
2-2	Applications of Proportions.	39
2-3	Precision and Accuracy.	45
	Ready to Go On? Module Quiz	52

MATHEMATICAL PRACTICES

The Common Core Georgia Performance Standards for Mathematical Practice describe varieties of expertise that all students should seek to develop. Opportunities to develop these practices are integrated throughout this program.

- 1 Make sense of problems and persevere in solving them.
- **2** Reason abstractly and quantitatively.
- **3** Construct viable arguments and critique the reasoning of others.
- 4 Model with mathematics.

- **5** Use appropriate tools strategically.
- 6 Attend to precision.
- 7 Look for and make use of structure.
- 8 Look for and express regularity in repeated reasoning.

Unpacking the Standards



Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this chapter.



Use units as a way to understand problems and to guide the solution of multi-step problems; ...

Key Vocabulary

unit analysis/dimensional analysis (análisis dimensional) A process that uses rates to convert measurements from one unit to another.

What It Means For You

Keeping track of units in problem solving will help you identify a solution method and interpret the results.

EXAMPLE

Li's car gets 40 miles per gallon of gas. At this rate, she can go 620 miles on a full tank. She has driven 245 miles on the current tank. How many gallons of gas g are left in the tank?





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Create equations ... in one variable and use them to solve problems.

Key Vocabulary

equation (ecuación)

A mathematical statement that two expressions are equivalent.

variable (variable)

A symbol used to represent a quantity that can change.



You can write an equation to represent a real-world problem and then use algebra to solve the equation and find the answer.

EXAMPLE

What It Means For You

Michael is saving money to buy a trumpet. The trumpet costs \$670. He has \$350 saved, and each week he adds \$20 to his savings. How long will it take him to save enough money to buy the trumpet?

Let *w* represent the number of weeks.

cost of trumpet		=	curre	ent sav	vings	+	additi	onal s	avings	
	670		=		350		+		20 <i>w</i>	
	320		=		20 <i>w</i>					
	16		=		w					

It will take Michael 16 weeks to save enough money.

Rates, Ratios, and Proportions



Essential Question: How can you use units to understand problems and guide the solution of proportions?

Objectives

Write and use ratios, rates, and unit rates.

2-1

Write and solve proportions.

Vocabulary

RF GPS

	-
ratio	proportion
rate	cross products
scale	scale drawing
unit rate	scale model
conversion	dimensional
factor	analysis

EXAMPLE

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Why learn this?

Ratios and proportions are used to draw accurate maps. (See Example 5.)

A **ratio** is a comparison of two quantities by division. The ratio of *a* to *b* can be written *a*:*b* or $\frac{a}{b}$, where $b \neq 0$. Ratios that name the same comparison are said to be *equivalent*.

A statement that two ratios are equivalent, such as $\frac{1}{12} = \frac{2}{24}$, is called a **proportion**.

Using Ratios

The ratio of faculty members to students at a college is 1:15. There are 675 students. How many faculty members are there?



Write a ratio comparing faculty to students.

Write a proportion. Let x be the number of faculty members.Since x is divided by 675, multiply both sides

Since x is divided by 675, multiply both sides of the equation by 675.

There are 45 faculty members.

Read the proportion $\frac{1}{15} = \frac{x}{675}$ as "1 is to

XAMPLE

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Reading Mat

15 as x is to 675."

ORE GPS

CHECK IT OUT!

1. The ratio of games won to games lost for a baseball team is 3:2. The team won 18 games. How many games did the team lose?

A **rate** is a ratio of two quantities with different units, such as $\frac{34 \text{ mi}}{2 \text{ gal}}$. Rates are usually written as *unit rates*. A **unit rate** is a rate with a second quantity of 1 unit, such as $\frac{17 \text{ mi}}{1 \text{ gal}}$, or 17 mi/gal. You can convert any rate to a unit rate.

Finding Unit Rates

IT OUT!

Takeru Kobayashi of Japan ate 53.5 hot dogs in 12 minutes to win a contest. Find the unit rate in hot dogs per minute. Round to the nearest hundredth.

$\frac{53.5}{12} = \frac{x}{1}$	Write a proportion to find an equivalent ratio with a second quantity of 1.			
$4.46 \approx x$	Divide on the left side to find x.			
The unit rate is approximately 4.46 hot dogs per minute.				

2. Cory earns \$52.50 in 7 hours. Find the unit rate in dollars per hour.



the two quantities are equal but use different units, is called a conversion factor. To convert from one set of units to another, multiply by a conversion factor. EXAMPL **Using Dimensional Analysis** MCC9-12.N.O.1 A large adult male human has about 12 pints of blood. Use dimensional analysis to convert this quantity to gallons. mv.hrw.com Step 1 Convert pints to quarts. $12 \text{ pt} \cdot \frac{1 \text{ qt}}{2 \text{ pt}}$ Multiply by a conversion factor whose first quantity is guarts and whose second quantity is pints. 6 qt 12 pints is 6 quarts. **Online Video Tuto** Step 2 Convert quarts to gallons. $6 \, \mathbf{qt} \cdot \frac{1 \, \mathrm{gal}}{4 \, \mathbf{qt}}$ Multiply by a conversion factor whose first quantity is gallons and whose second $\frac{6}{4}$ gal = $1\frac{1}{2}$ gal quantity is quarts. A large adult male human has about $1\frac{1}{2}$ gallons of blood. The dwarf sea horse Hippocampus zosterae swims at a rate of

Dimensional analysis is a process that uses rates to convert

measurements from one unit to another. A rate such as $\frac{12 \text{ in.}}{1 \text{ ft}}$, in which

52.68 feet per hour. Use dimensional analysis to convert this speed to inches per minute.

Use the conversion factor $\frac{12 \text{ in.}}{1 \text{ ft}}$ to convert feet to inches, and use the conversion factor $\frac{1 \text{ h}}{60 \text{ min}}$ to convert hours to minutes.

$$\frac{52.68 \text{ ft}}{1 \text{ h}} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} \cdot \frac{1 \text{ h}}{60 \text{ min}} = \frac{10.536 \text{ in.}}{1 \text{ min}}$$

The speed is 10.536 inches per minute.

Check that the answer is reasonable. The answer is about 10 in./min.

- There are 60 min in 1 h, so 10 in./min is 60(10) = 600 in./h.
- There are 12 in. in 1 ft, so 600 in./h is $\frac{600}{12} = 50$ ft/h. This is close to the rate given in the problem, 52.68 ft/h.

3. A cyclist travels 56 miles in 4 hours. Use dimensional analysis to convert the cyclist's speed to feet per second. Round your answer to the nearest tenth, and show that your answer is reasonable.

In the proportion $\frac{a}{b} = \frac{c}{d}$, the products $a \cdot d$ and $b \cdot c$ are called **cross products**. You can solve a proportion for a missing value by using the Cross Products Property.

Know it						
ante	WORDS	NUMBERS	ALGEBRA			
Mole	In a proportion, cross products are equal.	$\frac{2}{3} \neq \frac{4}{6}$	If $\frac{a}{b} \times \frac{c}{d}$ and $b \neq 0$ and $d \neq 0$,			
			then ad = bc .			



ORF GP

33



A scale is a ratio between two sets of measurements, such as 1 in:5 mi. A scale drawing or scale model uses a scale to represent an object as smaller or larger than the actual object. A map is an example of a scale drawing.



How many inches long is the actual heart it represents?







GUIDED PRACTICE

- Vocabulary What does it mean when two ratios form a *proportion*?
 The ratio of the cole price of a isolation the price of a isolation of the price of a second second
- **2.** The ratio of the sale price of a jacket to the original price is 3:4. The original price is \$64. What is the sale price?
- **3. Chemistry** The ratio of hydrogen atoms to oxygen atoms in water is 2:1. If an amount of water contains 341 trillion atoms of oxygen, how many hydrogen atoms are there?
- **4.** A computer's fan rotates 2000 times in 40 seconds. Find the unit rate in rotations per second.
- 5. Twelve cows produce 224,988 pounds of milk. Find the unit rate in pounds per cow.
- **6.** A yellow jacket can fly 4.5 meters in 9 seconds. Find the unit rate in meters per second.
- 7. Lydia wrote $4\frac{1}{2}$ pages of her science report in one hour. What was her writing rate in pages per minute?
- **8.** A model airplane flies 18 feet in 2 seconds. What is the airplane's speed in miles per hour? Round your answer to the nearest hundredth.
- **9.** A vehicle uses 1 tablespoon of gasoline to drive 125 yards. How many miles can the vehicle travel per gallon? Round your answer to the nearest mile. (*Hint*: There are 256 tablespoons in a gallon.)

SEE EXAMPLE 4

SEE EXAMPLE

SEE EXAMPLE

SEE EXAMPLE 3

Solve each proportion.

10.
$$\frac{3}{z} = \frac{1}{8}$$
11. $\frac{x}{3} = \frac{1}{5}$
12. $\frac{b}{4} = \frac{3}{2}$
13. $\frac{f+3}{12} = \frac{7}{2}$
14. $\frac{-1}{5} = \frac{3}{2d}$
15. $\frac{3}{14} = \frac{s-2}{21}$
16. $\frac{-4}{9} = \frac{7}{x}$
17. $\frac{3}{s-2} = \frac{1}{7}$
18. $\frac{10}{h} = \frac{52}{13}$

SEE EXAMPLE 5 19. Archaeology Stonehenge II in Hunt, Texas, is a scale model of the ancient construction in Wiltshire, England. The scale of the model to the original is 3:5. The Altar Stone of the original construction is 4.9 meters tall. Write and solve a proportion to find the height of the model of the Altar Stone.

Alfred Sheppard, one of the builders of Stonehenge II.

PRACTICE AND PROBLEM SOLVING

- **20.** Gardening The ratio of the height of a bonsai ficus tree to the height of a full-size ficus tree is 1:9. The bonsai ficus is 6 inches tall. What is the height of a full-size ficus?
- **21. Manufacturing** At one factory, the ratio of defective light bulbs produced to total light bulbs produced is about 3:500. How many light bulbs are expected to be defective when 12,000 are produced?
- **22.** Four gallons of gasoline weigh 25 pounds. Find the unit rate in pounds per gallon.
- **23.** Fifteen ounces of gold cost \$6058.50. Find the unit rate in dollars per ounce.
- 24. Biology The tropical giant bamboo can grow 11.9 feet in 3 days. What is this rate of growth in inches per hour? Round your answer to the nearest hundredth, and show that your answer is reasonable.
- 25. Transportation The maximum speed of the Tupolev Tu-144 airliner is 694 m/s. What is this speed in kilometers per hour?

Solve each proportion.

- **26.** $\frac{v}{6} = \frac{1}{2}$ **27.** $\frac{2}{5} = \frac{4}{y}$ **28.** $\frac{2}{h} = \frac{-5}{6}$ **29.** $\frac{3}{10} = \frac{b+7}{20}$ **30.** $\frac{5t}{9} = \frac{1}{2}$ **31.** $\frac{2}{3} = \frac{6}{q-4}$ **32.** $\frac{x}{8} = \frac{7.5}{20}$ **33.** $\frac{3}{k} = \frac{45}{18}$ **34.** $\frac{6}{a} = \frac{15}{17}$ **35.** $\frac{9}{2} = \frac{5}{x+1}$ **36.** $\frac{3}{5} = \frac{x}{100}$
- **38.** Science The image shows a dust mite as seen under a microscope. The scale of the drawing to the dust mite is 100:1. Use a ruler to measure the length of the dust mite in the image in millimeters. What is the actual length of the dust mite?
- **39.** Finance On a certain day, the exchange rate was 60 U.S. dollars for 50 euro. How many U.S. dollars were 70 euro worth that day? Show that your answer is reasonable.
- Environmental Science An environmental scientist wants to estimate the number of carp in a pond. He captures 100 carp, tags all of them, and releases them. A week later, he captures 85 carp and records how many have tags. His results are shown in the table. Write and solve a proportion to estimate the number of carp in the pond.

10 20
33.
$$\frac{3}{k} = \frac{45}{18}$$

37. $\frac{38}{19} = \frac{n-5}{20}$

Independent Practice			
For Exercises	See Example		
20–21	1		
22–23	2		
24–25	3		
26–37	4		
38	5		





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1	0
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	2
1	
4	2

41. *[]* **[ERROR ANALYSIS** *[]* Below is a bonus question that appeared on an algebra test and a student's response.

The ratio of junior varsity members to varsity3members on the track team is 3:5. There are524 members on the team. Write a proportion5to find the number of junior varsity members.

The student did not receive the bonus points. Why is this proportion incorrect?

Sports The table shows world record times for women's races of different distances.

- **a.** Find the speed in meters per second for each race. Round your answers to the nearest hundredth.
- b. Which race has the fastest speed? the slowest?
- **c. Critical Thinking** Give a possible reason why the speeds are different.

World Records (Women)		
Distance (m)	Time (s)	
100	10.5	
200	21.3	
800	113.3	
5000	864.7	

 $=\frac{x}{24}$

43. Entertainment Lynn, Faith, and Jeremy are film animators. In one 8-hour day, Lynn rendered 203 frames, Faith rendered 216 frames, and Jeremy rendered 227 frames. How many more frames per hour did Faith render than Lynn did?

Solve each proportion.

44.	$\frac{x-1}{3} = \frac{x+1}{5}$	45. $\frac{m}{3} = \frac{m+4}{7}$	46. $\frac{1}{x-3} = \frac{3}{x-5}$	47. $\frac{a}{2} = \frac{a-4}{30}$
48.	$\frac{3}{2y} = \frac{16}{y+2}$	49. $\frac{n+3}{5} = \frac{n-1}{2}$	50. $\frac{1}{y} = \frac{1}{6y - 1}$	51. $\frac{2}{n} = \frac{4}{n+3}$
52.	$\frac{5t-3}{-2} = \frac{t+3}{2}$	53. $\frac{3}{d+3} = \frac{4}{d+12}$	54. $\frac{3x+5}{14} = \frac{x}{3}$	55. $\frac{5}{2n} = \frac{8}{3n-24}$

- **56. Decorating** A particular shade of paint is made by mixing 5 parts red paint with 7 parts blue paint. To make this shade, Shannon mixed 12 quarts of blue paint with 8 quarts of red paint. Did Shannon mix the correct shade? Explain.
- **HOT** 57. Write About It Give three examples of proportions. How do you know they are proportions? Then give three nonexamples of proportions. How do you know they are not proportions?

Real-World Connections

- **58. a.** Marcus is shopping for a new jacket. He finds one with a price tag of \$120. Above the rack is a sign that says that he can take off $\frac{1}{5}$. Find out how much Marcus can deduct from the price of the jacket.
 - b. What price will Marcus pay for the jacket?
 - **c.** Copy the model below. Complete it by placing numerical values on top and the corresponding fractional parts below.



- d. Explain how this model shows proportional relationships.

(cl), C Duomo/CORBIS; (bl), PhotoDisc/gettyimages



The records for the women's 100-meter dash and the women's 200-meter dash were set by Florence Griffith-Joyner, known as "Flo Jo." She is still referred to as the world's fastest woman.

TEST PREP

59. One day the U.S. dollar was worth approximately 100 yen. An exchange of 2500 yen was made that day. What was the value of the exchange in dollars?

(A) \$25
(B) \$400
(C) \$2500
(D) \$40,000

60. Brett walks at a speed of 4 miles per hour. He walks for 20 minutes in a straight line at this rate. Approximately what distance does Brett walk?

(F) 0.06 miles (G) 1.3 miles (H) 5 miles (J) 80 miles

61. A shampoo company conducted a survey and found that 3 out of 8 people use their brand of shampoo. Which proportion could be used to find the expected number of users *n* in a city of 75,000 people?

(A)
$$\frac{3}{8} = \frac{75,000}{n}$$
 (B) $\frac{3}{75,000} = \frac{n}{8}$ (C) $\frac{8}{3} = \frac{n}{75,000}$ (D) $\frac{3}{8} = \frac{n}{75,000}$

62. A statue is 3 feet tall. The display case for a model of the statue can fit a model that is no more than 9 inches tall. Which of the scales below allows for the tallest model of the statue that will fit in the display case?

(F) 2:1 (G) 1:1 (H) 1:3 (J) 1:4

CHALLENGE AND EXTEND

- **63. Geometry** Complementary angles are two angles whose measures add up to 90°. The ratio of the measures of two complementary angles is 4:5. What are the measures of the angles?
- **64.** A customer wanted 24 feet of rope. The clerk at the hardware store used what she thought was a yardstick to measure the rope, but the yardstick was actually 2 inches too short. How many inches were missing from the customer's piece of rope?
- **65. Population** The population density of Jackson, Mississippi, is 672.2 people per square kilometer. What is the population density in people per square meter? Show that your answer is reasonable. (*Hint:* There are 1000 meters in 1 kilometer. How many square meters are in 1 square kilometer?)

MATHEMATICAL

FOCUS ON MATHEMATICAL PRACTICES

HOT 66. Error Analysis Sofia says that any real number is a solution to the equation $\frac{4}{2x-4} = \frac{2}{x-2}$. What mistake did she make?

HOT 67. Make a Conjecture Examine the graph.

- **a.** Do the two points on Line 1 satisfy the proportion $\frac{y_1}{x_1} = \frac{y_2}{x_2}$? Explain.
- **b.** Do the two points on Line 2 satisfy the proportion $\frac{y_1}{x_1} = \frac{y_2}{x_2}$? Explain.
- **c.** Another point on Line 1 is (1, 3). Replace one of the points from part a with this point. Do these two points satisfy the proportion?
- **d.** Another point on Line 1 is (6, 4). Replace one of the points from part b with this point. Do these two points satisfy the proportion?
- **e.** Make a conjecture about whether the coordinates of any two points on each line will form a proportion.

10 8 line (9.6) 6 (2, 4)Δ Line 2 2 (3, 2)х 0 8 10 Δ 6

HOT. 68. Problem Solving Find a solution of $\frac{12}{x} = \frac{x}{3}$. Explain how you found it.

Applications of Proportions



Essential Question: How can you create proportions and use them to solve problems?

Objectives

Use proportions to solve problems involving geometric figures.

2-2

Use proportions and similar figures to measure objects indirectly.

Vocabulary

similar corresponding sides corresponding angles indirect measurement scale factor



Reading Math 📷

- AB means segment AB. AB means the length of AB.
- ∠A means angle A. m∠A means the measure of angle A.

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EXAMPLE

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Proportions can be used to find the heights of tall objects, such as totem poles, that would otherwise be difficult to measure. (See Example 2.)

Similar figures have exactly the same shape but not necessarily the same size.

Corresponding sides of two figures are in the same relative position, and **corresponding angles** are in the same relative position. Two figures are similar if and only if the lengths of corresponding sides are proportional and all pairs of corresponding angles have equal measures.





When stating that two figures are similar, use the symbol \sim . For the triangles above, you can write $\triangle ABC \sim \triangle DEF$. Make sure corresponding vertices are in the same order. It would be incorrect to write $\triangle ABC \sim \triangle EFD$.

You can use proportions to find missing lengths in similar figures.

Finding Missing Measures in Similar Figures

Find the value of *x* in each diagram.

 $\triangle RST \sim \triangle BCD$



R corresponds to *B*, *S* corresponds to *C*, and *T* corresponds to *D*.

$\frac{5}{12}$	=	$\frac{8}{x}$
5x	=	96
5 <i>x</i>	_	96
5	_	5
x	=	19.2

 $\frac{RT}{BD} = \frac{RS}{BC}$ Use cross products. Since x is multiplied by 5, divide both sides by 5 to undo the multiplication.

The length of \overline{BC} is 19.2 ft.

Robert Harding Picture Library Ltd/Alamy



You can solve a proportion involving similar triangles to find a length that is not easily measured. This method of measurement is called **indirect measurement**. If two objects form right angles with the ground, you can apply indirect measurement using their shadows.

Measurement Application

A totem pole casts a shadow 45 feet long at the same time that a 6-foot-tall man casts a shadow that is 3 feet long. Write and solve a proportion to find the height of the totem pole.





2a. A forest ranger who is 150 cm tall casts a shadow 45 cm long. At the same time, a nearby tree casts a shadow 195 cm long. Write and solve a proportion to find the height of the tree.

2b. A woman who is 5.5 feet tall casts a shadow 3.5 feet long. At the same time, a building casts a shadow 28 feet long. Write and solve a proportion to find the height of the building.

Helpful Hint

A height of 90 ft seems reasonable for a totem pole. If you got 900 or 9000 ft, that would not be reasonable, and you should check your work.

EXAMPLE

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DMMON

DRE GPS

If every dimension of a figure is multiplied by the same number, the result is a similar figure. The multiplier is called a scale factor.



Helpful Hint

A scale factor between 0 and 1 reduces a figure. A scale factor greater than 1 enlarges it.

Changing Dimensions

Every dimension of a 2-by-4-inch rectangle is multiplied by 1.5 to form a similar rectangle. How is the ratio of the perimeters related to the ratio of corresponding sides? How is the ratio of the areas related to the ratio of corresponding sides?



	Rectangle A	Rectangle B	
$P=2\ell+2w$	2(2) + 2(4) = 12	2(6) + 2(3) = 18	
$A = \ell w$	4(2) = 8	6(3) = 18	
Sides: $\frac{4}{6} = \frac{2}{3}$	Perimeters	$s: \frac{12}{18} = \frac{2}{3}$	

Perimeters: $\frac{12}{18} = \frac{2}{3}$ Areas: $\frac{8}{18} = \frac{4}{9} = \left(\frac{2}{3}\right)^2$

The ratio of the perimeters is equal to the ratio of corresponding sides. The ratio of the areas is the square of the ratio of corresponding sides.

Every dimension of a cylinder with radius 4 cm and height 6 cm is multiplied by $\frac{1}{2}$ to form a similar cylinder. How is the ratio of the volumes related to the ratio of corresponding dimensions?



Cylinder ACylinder B
$$V = \pi r^2 h$$
 $\pi(4)^2(6) = 96\pi$ $\pi(2)^2(3) = 12\pi$ Radii: $\frac{4}{2} = \frac{2}{1} = 2$ Heights: $\frac{6}{3} = \frac{2}{1} = 2$ Volumes: $\frac{96\pi}{12\pi} = \frac{8}{1} = 8 = 2^3$

The ratio of the volumes is the cube of the ratio of corresponding dimensions.



3. A rectangle has width 12 inches and length 3 inches. Every dimension of the rectangle is multiplied by $\frac{1}{3}$ to form a similar rectangle. How is the ratio of the perimeters related to the ratio of the corresponding sides?



2-2 Exercises





PRACTICE AND PROBLEM SOLVING

Independent Practice				
For Exercises	See Example			
6–7	1			
8	2			
9	3			



Find the value of *x* in each diagram.

the ratio of corresponding sides?



- **8.** Write and solve a proportion to find the height of the taller tree in the diagram at right.
- **9.** A triangle has side lengths of 5 inches, 12 inches, and 15 inches. Every dimension is multiplied by $\frac{1}{5}$ to form a new triangle. How is the ratio of the perimeters related to the ratio of corresponding sides?
- **10. Hobbies** For a baby shower gift, Heather crocheted a baby blanket whose length was $2\frac{1}{2}$ feet and whose width was 2 feet. She plans to crochet a proportionally larger similar blanket for the baby's mother. If she wants the length of the mother's blanket to be $6\frac{1}{4}$ feet, what should the width be? Show that your answer is reasonable.

multiplied by $\frac{3}{4}$ to form a similar rectangle. How is the ratio of the areas related to

- **11. Real Estate** Refer to the home builder's advertisement. The family rooms in both models are rectangular. How much carpeting is needed to carpet the family room in the Weston model?
- **12.** A rectangle has an area of 16 ft². Every dimension is multiplied by a scale factor, and the new rectangle has an area of 64 ft². What was the scale factor?



13. A cone has a volume of 98π cm³. Every dimension is multiplied by a scale factor, and the new cone has a volume of 6272π cm³. What was the scale factor?

Find the value of *x* in each diagram.



- **18.** A tower casts a 450 ft shadow at the same time that a 4 ft child casts a 6 ft shadow. Write and solve a proportion to find the height of the tower.
- **HOT** 19. Write About It At Pizza Palace, a pizza with a diameter of 8 inches costs \$6.00. The restaurant manager says that a 16-inch pizza should be priced at \$12.00 because it is twice as large. Do you agree? Explain why or why not.
 - **20.** Another common application of proportion is *percents*. A percent is a ratio of a number to 100. For example, $80\% = \frac{80}{100}$.
 - **a.** Write 12%, 18%, 25%, 67%, and 98% as ratios.
 - **b.** Percents can also be written as decimals. Write each of your ratios from part **a** as a decimal.
 - c. What do you notice about a percent and its decimal equivalent?

Real-World

Connections

- **21.** A lighthouse casts a shadow that is 36 meters long. At the same time, a person who is 1.5 meters tall casts a shadow that is 4.5 meters long. Write and solve a proportion to find the height of the lighthouse.
- **22.** In the diagram, $\triangle ABC \sim \triangle DEC$. What is the distance across the river from *A* to *B*?
- **HOT 23. Critical Thinking** If every dimension of a two-dimensional figure is multiplied by *k*, by what quantity is the area multiplied?



TEST PREP

- **24.** A beach ball holds 800 cubic inches of air. Another beach ball has a radius that is half that of the larger ball. How much air does the smaller ball hold?
 - A 400 cubic inches
 C 100 cubic inches
 - B 200 cubic inches
 D 80 cubic inches
- **25.** For two similar triangles, $\frac{SG}{MW} = \frac{GT}{WR} = \frac{TS}{RM}$. Which statement below is NOT correct?
 - $\textcircled{F} \quad \triangle SGT \sim \triangle MWR \qquad \qquad \textcircled{H} \quad \triangle TGS \sim \triangle RWM$
- **26. Gridded Response** A rectangle has length 5 centimeters and width 3 centimeters. A similar rectangle has length 7.25 centimeters. What is the width in centimeters of this rectangle?

CHALLENGE AND EXTEND

- **27.** Find the values of *w*, *x*, and *y* given that $\triangle ABC \sim \triangle DEF \sim \triangle GHJ$.
- **28.** $\triangle RST \sim \triangle VWX \text{ and } \frac{RT}{VX} = b.$ What is $\frac{\text{area of } \triangle RST}{\text{area of } \triangle VWX}$?
- **29. Multi-Step** Rectangles *A* and *B* are similar. The area of *A* is 30.195 cm². The length of *B* is 6.1 cm. Each dimension of *B* is $\frac{2}{3}$ the corresponding dimension of *A*. What is the perimeter of *B*?



MATHEMATICAL

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- **HOT 30. Modeling** It takes Padma 6 minutes to cut a length of timber into 3 pieces. How long would it take her to cut a length into 9 pieces? (*Hint:* Think about how many *cuts* it takes to cut the timber into 3 pieces or 9 pieces.)
- **HOT** 31. Problem Solving You have a stack of $8\frac{1}{2}$ in. wide by 11 in. long sheets of paper, and start laying the sheets out as shown. The shape is the same number of sheets wide as it is long.
 - a. When the shape is 68 in. wide, how long is it?
 - **b.** When the area of the shape is 3366 in.², how many sheets are in it?



Precision and Accuracy

Essential Question: How can you choose appropriate levels of precision and accuracy when solving problems?

Objectives

Analyze and compare measurements for precision and accuracy.

2-3

Choose an appropriate level of accuracy when reporting measurements.

Vocabulary

precision accuracy tolerance

Who uses this?

Chemists must understand precision and accuracy when weighing or mixing specific amounts of chemicals. (See Example 2.)

When you measure an object, you must use an instrument that will give an appropriate measurement. A scale to measure the mass of a person may show mass to the nearest kilogram. A scale to measure chemicals in a lab may show mass to the nearest milligram.

Precision is the level of detail in a measurement and is determined by the smallest unit or fraction of a unit that you can reasonably measure. Sometimes, the instrument determines the precision of a measurement. At other times, measurements are rounded to a specified precision.



A scale that shows the mass of an object to the nearest milligram is more precise than a scale that shows the mass of an object to the nearest kilogram, because a milligram is a smaller unit of measure than a kilogram. Likewise, a scale that shows the mass of an object as 24.23 grams is more precise than a scale that shows the mass of the same object as 24.2 grams.





A precise measurement is only useful if the measurement is also *accurate*. The **accuracy** of a measurement is the closeness of a measured value to the actual or true value. Two measurement tools may measure to the same precision, but not have the same accuracy. Similarly, using a more precise measuring instrument will not necessarily give a more accurate measurement.



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Comparing Precision and Accuracy

Sam is a technician in a pharmaceutical lab. Each week, she must test the scales in the lab to make sure they are accurate. She uses a standard mass that is *exactly* 5.000 grams and gets the following results:



a. Which scale is the most precise?

Scales 1 and 3 measure to the nearest hundredth of a gram.

Scale 2 measures to the nearest thousandth of a gram.

Because a thousandth of a gram is smaller than a hundredth of a gram, Scale 2 is the most precise.

b. Which scale is the most accurate?

For each scale, find the absolute value of the difference of the standard mass and the scale reading.

Scale 1: |5.000 - 5.01| = 0.01Scale 2: |5.000 - 5.033| = 0.033

|5000 - 5000| = 0.050

Scale 3: |5.000 - 4.98| = 0.02

Because 0.01 < 0.02 < 0.033, Scale 1 is the most accurate.



2. A standard mass of 16 ounces is used to test three postal scales. The results are shown below.



- **a.** Which scale is the most precise?
- **b.** Which scale is the most accurate?

When you measure a group of objects that are expected to be similar, you may find that there are variations from the expected value. **Tolerance** describes the amount by which a measurement is permitted to vary from a specified value. Tolerance is often expressed as a range of values, such as 5 mm \pm 0.3 mm, which is equivalent to 4.7 mm–5.3 mm.



Writing Math

The final zero in a

dropped. 50.0 mm

of one-tenth of a millimeter. 50 mm

than 50.0 mm.

indicates a precision

indicates a precision of one millimeter, and is less precise

such as 50.0 mm

should not be

decimal measurement



Using a Specified Tolerance

Acme Nuts & Bolts is manufacturing a bolt to use in an airplane. The length of the bolt should be 50 mm, with a tolerance of 0.5 mm (50 mm \pm 0.5 mm). A batch of bolts had the lengths shown in the table. Do all of the bolts measure within the specified tolerance? If not, which bolt(s) are not within the specified tolerance?

	Bolt	Length (mm)
9	Ф A	49.8
-	В	50.4
	C	49.5
	D	50.1
	E	49.4
	F	50.0

50-0.5 = 49.5 50 mm \pm 0.5 mm means that the 50+0.5 = 50.5 bolts must be between 49.5 and 50.5 mm.

Bolt E measures 49.4 mm, so it is not within the specified tolerance.



3. A lacrosse ball must weigh $5.25 \text{ oz} \pm 0.25 \text{ oz}$. The weights of the lacrosse balls in one box are given in the table. Do all of the lacrosse balls weigh within the specified tolerance? If not, which lacrosse ball(s) are not within the specified tolerance?

Ball	Weight (oz)
А	5.41
В	5.23
C	5.54
D	5.33
E	5.21

Tolerance can also be expressed as a percent. A measurement written as $5 \text{ mm} \pm 5\%$ means that the measurement can be greater or less than 5 mm by an amount equal to 5% of 5 mm, or 0.25 mm. Therefore, the measurement can have a range of 4.75 mm–5.25 mm.

COMMON CORE GPS	EXAMPLE MCC9-12.N.Q.2	4	Using Tolerance Expressed as a Percent				
imy.hrw.com		Write the possible range of each measurement. Round to the nearest hundredth if necessary.					
			$A 50 \text{ kg} \pm 2\%$				
			50(0.02) = 1	Find 2% of 50.			
	200		50 kg ± 1 kg	Write the measurement and tolerance.			
			49 kg–51 kg	Write the measurement as a range.			
	THE REPORT OF A PARTY OF A		B 125 lb \pm 1.5%				
	Online Video Tutor		125(0.015) = 1.875	Find 1.5% of 125.			
			$125 \text{ lb} \pm 1.88 \text{ lb}$	Write the measurement and tolerance. Round to the nearest hundredth.			
			123.12 lb–126.88 lb	Write the measurement as a range.			
			C 45 mm ± 0.3%				
			45(0.003) = 0.135	Find 0.3% of 45.			
			45 mm ± 0.14 mm	Write the measurement and tolerance. Round to the nearest hundredth.			
			44.86 mm–45.14 mm	Write the measurement as a range.			

Write the possible range of each measurement. Round to the nearest hundredth if necessary.

4a. 4.1 in. \pm 5% **4b.** 475 m \pm 2.5% **4c.** 85 mg \pm 0.5%





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GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.

- 1. A ruler that can measure length to a smaller unit than another ruler is said to be more _____? ___. (*precise* or *acurate*)
- **2.** A scale that gives a mass closer to the true mass of an object than another scale of the exact same type is said to be more ______. (*precise* or *accurate*)

S	E	E	E	X	A	Μ	P	L	E	1	1

Choose the more precise measurement in each pair.

3.	4 mL; 4.3 mL	4. 7 m; 6.8 m	5.	2.4 mg; 2.37 mg
6.	7 lb; 6.5 lb	7. 47 ft; 47.3 ft	8.	14 oz; 13.9 oz

SEE EXAMPLE 2

9. Sarah is comparing five different scales using a standard mass that is exactly 10 grams. Her results are shown below.



- a. Which scale is the most precise?
- **b.** Which scale is the most accurate?
- **10.** A group of students compare the odometer readings on their bicycle computers after riding their bikes on a one-mile track. Their odometer readings are shown in the table. Whose odometer is the most precise? Whose is the most accurate?

Student	Distance (mi)
Jen	1.01
Bill	0.97
Rasheed	0.989
Sasha	1.02

SEE EXAMPLE 3

11. Sports A basketball for men's college games must have a mass of 595.5 ± 28.5 grams. Several basketballs are tested. Their masses are shown in the table. Do all of the basketballs fall within the specified tolerance? If not, which basketball(s) do not fall within the specified tolerance?

Basketball	1	2	3	4	5
Mass (g)	617.5	567.5	608	624.5	593.5

12. Sports A basketball for men's college games must bounce 51.5 ± 2.5 in. when dropped from a height of 6 feet. The bounce heights of several basketballs when dropped from a height of 6 feet are shown in the graph. Do all of the basketballs fall within the specified tolerance? If not, which basketball(s) do not have a bounce height within the specified tolerance?



SEE EXAMPLE 4

Write the possible range of each measurement. Round to the nearest hundredth if necessary.

13. 50 lb ± 2%	14. 100 yd \pm 0.5%	15. 25 cm ± 4%
16. 400 L ± 6%	17. 250 mm ± 4%	18. 70 kg ± 3%

PRACTICE AND PROBLEM SOLVING

Independer	nt Practice	Ch
For Exercises	See Example	19
19–26	1	23
27	2	
28	3	27
29-36	4	



Cho	Choose the more precise measurement in each pair.						
19.	4.33 g; 4337 mg	20. 11 ft; 122 in.	21. 6 tons; 11,000 lb	22. 3 c; 2 pt			
23.	67 mm; 6.83 cm	24. 4.5 km; 3 mi	25. 12 cm; 0.0127 m	26. 7.23 lb; 115 oz			

- 7. Maria is trying to beat the school record for the 400-meter dash. Her friends timed her using the stopwatch functions in their cell phones. The official track timer, which is highly accurate, reported that she ran the race in 51.12 seconds. Her friends recorded the times shown in the table.
 - **a.** Who recorded the most precise time?
 - **b.** Who recorded the most accurate time?
- **28.** Anael cut several boards to build a deck. The boards must be 100 in. \pm 0.25 in. Her measurements of the boards after cutting them are shown in the graph. Which boards, if any, can she not use?

Name	Time (s)
Lucy	51.1
Juan	52.23
Chandra	51.769
Pei	50.97



Write the possible range of each measurement. Round to the nearest hundredth if necessary.

29.	$45 \text{ lb} \pm 2\%$	30. 3 m ± 5%	31. 37 °C ± 1.5%	32. 750 kg ± 3%
33.	30 ft ± 4%	34. 550 mL ± 8%	35. 0.2 cm ± 5%	36. $0.25 \text{ kg} \pm 10\%$

Round each measurement to the specified precision.

- **37.** 5456.3 mi to the nearest mile
- 38. 3.627 m to the nearest hundredth of a meter
- **39.** 119.8 ft to the nearest ten feet
- **40.** 62.301 cg to the nearest tenth of a centigram
- **41.** 5,721 mg to the nearest kilogram
- 42. 0.4586 km to the nearest meter

Choose the more precise measurement in each pair. If they are equally precise, write "neither."

43.	16.270 liters; 16,453.2 mL	44.	437 cm; 437 mm	45. 0.265 cm; 260 mm
46.	5.20 kg; 5200.0 mg	47.	55 yd; 165 ft	48. 67 min; 1.1 h
49.	33 mg; 0.033 g	50.	42.7 cm; 427.0 mm	51. 475.0 mL; 0.475 L

Rewrite each specified tolerance as a percent.

52.	$100 \text{ m} \pm 2 \text{ m}$	53. 50 g ± 2 g	54. 240 ft ± 12 ft	55.	750 kg ± 15 kg
56.	$25 \text{ in.} \pm 0.25 \text{ in.}$	57. 425 lb ± 8.5 lb	58. 60 oz ± 1.5 oz	59.	175 km ± 5.25 km

Technology Postcards that do not fit in the U.S. Postal Service's automatic sorting machines require additional postage for mailing. The machine will accept postcards whose length is between 5 and 6 inches and whose width is between $3\frac{1}{2}$ and $4\frac{1}{4}$ inches. Write these requirements as tolerances.

©Steve Hamblin/Alamy

61. Sports For women's collegiate competition, a basketball's circumference, mass, and bounce height must fall within given tolerance levels of regulation measurements. The table shows these tolerance levels as well as measurements taken on five different basketballs. Which basketball meets all of the specified tolerances?

	Circumference (mm)	Mass (g)	Bounce Height (mm)
Tolerance	730.56 ± 6.5	538.5 ± 28.5	1358.5 ± 63.5
Basketball #1	729.8	509.3	1343.4
Basketball #2	723.5	529.8	1299.8
Basketball #3	734.2	542.6	1293.5
Basketball #4	725.5	528.0	1364.5
Basketball #5	740.0	555.9	1407.4

- **HOT 62.** Write About It Linda wants to purchase a new sofa. Before buying the sofa, Linda must measure her doorway to make sure that the sofa will fit through the door. The sofa manufacturer says that the sofa measures 39 inches from front to back. What level of precision would you recommend Linda measure to? Explain.
- **HOT 63.** Critical Thinking Yusuf measured a board and determined that it was 125.5 centimeters long. He then cut the board into eight equal pieces. His calculator shows that $125.5 \div 8 = 15.6875$. Is it reasonable for Yusuf to record the length of the 8 smaller boards as 15.6875 centimeters? Explain why or why not.



Automated equipment plays a large role in processing the approximately 584 million pieces of mail that the U.S. Postal Service delivers each day. Machines sort mail, cancel stamps, scan barcodes, and even "read" handwritten addresses.

Source: Postal Facts 2010, USPS

TEST PREP

- 64. The mass of a crystal is 0.9728 grams. What is the mass of the crystal to the nearest milligram?
 - (A) 1 milligram

- © 973 milligrams
- **B** 9.73 milligrams

- **D** 972.8 milligrams
- **65.** A piece used to assemble a computer must be 1.4 millimeters \pm 0.02 millimeters in diameter. Which of the following measurements does NOT meet the specified tolerance?
 - (F) 1.420 millimeters (H) 1.382 millimeters
 - G 1.402 millimeters (J) 1.378 millimeters
- 66. Which measurement is most precise?
 - (A) 475.3 milliliters **(C)** 0.475 liter
 - **B** 475 milliliters **D** 0.5 liter

CHALLENGE AND EXTEND

Percent accuracy or percent error indicates how far a measurement is from the true value. An instrument that has 1.5% accuracy means that the measured value is within 1.5% of the true value.

- **67.** A scale shows that a standard mass of exactly 5.000 grams has a mass of 5.002 grams. What is the percent accuracy of the scale?
- **68.** A car odometer is accurate to within 0.5%. The odometer records the distance from Charlotte, North Carolina, to Orlando, Florida, as 525.3 miles. What is the range of possible values for the actual mileage?
- **69.** Astronomy A scientist measures the distance to the moon using a method that has a percent error of 0.02%. He finds that the distance at a particular time is 384,403 kilometers. What is the range of possible values for the actual distance?



FOCUS ON MATHEMATICAL PRACTICES

- **HOT** 70. **Problem Solving** An Internet sports site polled its readers with the question "Which team will win the division?" and posted the results. What is the smallest number of readers that could have picked Atlanta? Explain your answer.
- **HOT 71. Communication** Would you prefer to have an accurate room thermometer that is not very precise or a precise thermometer that is not very accurate? Explain.
- **HOT** 72. Error Analysis Caleb uses the ruler shown to measure the length of a card. He says that the length is 3.1875 inches, so the measurement is precise to one ten-thousandth of an inch. Is he correct? Explain.

Atlanta	33%
Tampa	29%
New Orleans	24%
Carolina	14%

3651 Responses



MODULE 2 QUIZ

Ready to Go On?

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⊘ 2

2-1 Rates, Ratios, and Proportions

- **1.** Last week, the ratio of laptops to desktops sold at a computer store was 2:3. Eighteen desktop models were sold. How many laptop models were sold?
- 2. Anita read 150 pages in 5 hours. What is her reading rate in pages per minute?
- **3.** Twenty-six crackers contain 156 Calories. Find the unit rate in Calories per cracker.
- 4. A store developed 1024 photographs in 8 hours. Find the unit rate in photographs per hour.

Solve each proportion.

5.
$$\frac{-18}{n} = \frac{9}{2}$$

6. $\frac{d}{5} = \frac{2}{4}$
7. $\frac{4}{12} = \frac{r+2}{16}$
8. $\frac{-3}{7} = \frac{6}{x+6}$

()

2-2 Applications of Proportions

Find the value of *n* in each diagram.

9. $\triangle RST \sim \triangle XYZ$

10. *ABCD* ~ *FGHJ*



Ø

2-3 Precision and Accuracy

Choose the more precise measurement in each pair.

11.	2.5 ft; 2 ft	12.	1 yd; 3 ft
13.	5910 g; 5.9 kg	14.	16 oz; 16.0 oz

Write the possible range of each measurement. Round to the nearest hundredth if necessary.

15.	$300 \text{ m} \pm 1\%$	16.	$150 \text{ lb} \pm 6\%$
17.	$60~\mathrm{L}\pm0.5\%$	18.	$220~\text{kg}\pm1.5\%$

PARCC Assessment Readiness



Selected Response

- The fuel for a chain saw is a mix of oil and gasoline. The ratio of ounces of oil to gallons of gasoline is 7:19. There are 38 gallons of gasoline. How many ounces of oil are there?
 - A 14 ounces
 - B 20 ounces
 - **(C)** 103.1 ounces
 - D 3.5 ounces
- **2.** A pipe is leaking at the rate of 8 fluid ounces per minute. Use dimensional analysis to find out how many gallons the pipe is leaking per hour.
 - **(F)** 3,840 gal/h
 - G 0.02 gal/h
 - (H) 3.75 gal/h
 - J 17.07 gal/h
- **3.** Solve the proportion $\frac{5}{6} = \frac{x}{30}$.
 - (A) *x* = 0.03
 - **B** *x* = 36
 - **(C)** *x* = 26
 - **D** *x* = 25
- **4.** Find the value of *MN* if *AB* = 21 cm, *BC* = 16.8 cm, and *LM* = 28 cm. *ABCD* ~ *LMNO*



5. Complementary angles are two angles whose measures add to 90°. The ratio of the measures of two complementary angles is 4:11. What are the measures of the angles?

A 24°, 66°	C 51.4°, 38.6°
B 26°, 64°	D 24°, 114°

- 6. A weight that measures *exactly* 3.000 ounces is placed on three different balance scales. Scale 1 shows a weight of 3.03 ounces, scale 2 shows a weight of 2.99 ounces, and scale 3 shows a weight of 3.014 ounces. Which scale is the most precise? Which is the most accurate?
 - (F) Scale 1 is the most precise. Scale 3 is the most accurate.
 - G Scale 3 is the most precise. Scale 2 is the most accurate.
 - (H) Scale 1 is the most precise. Scale 2 is the most accurate.
 - Scale 3 is the most precise.Scale 3 is the most accurate.
- Round the measurement and underline the last significant digit.
 254.8 liters to the nearest liter.
 - (A) 254 liters
 - (B) 260 liters
 - C 25<u>5</u> liters
 - **D** 2<u>5</u>0 liters
- Write the possible range of the measurement to the nearest hundredth.
 40 km + 1%
 - (F) 39.8 km—40.2 km
 - (G) 39.99 km-40.01 km
 - (H) 39 km—41 km
 - J 39.6 km—40.4 km

Mini-Task

- **9.** A recipe for a casserole calls for 2 cups of rice. The recipe makes 6 servings of casserole.
 - a. How many cups of rice will you need to make 10 servings of casserole?
 - **b.** If you have 5 cups of rice, how many servings can you make?