Module

4

Solving Inequalities in One Variable

COMMON CORE GPS

Contents

MATHEMATICAL PRACTICES   The Common Core Georgia Performance Standards for Mathematical Practice describe varieties of expertise that all students should seek to develop. Opportunities to develop these practices are integrated throughout this program.

1 Make sense of problems and persevere in solving them.

2 Reason abstractly and quantitatively.

3 Construct viable arguments and critique the reasoning of others.

4 Model with mathematics.

5 Use appropriate tools strategically.

6 Attend to precision.

7 Look for and make use of structure.

8 Look for and express regularity in repeated reasoning.

Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this chapter.

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Represent constraints by … inequalities … and interpret solutions as viable or nonviable options in a modeling context.

Key Vocabulary

inequality (desigualdad) A statement that compares two expressions by using one of the following signs: $<, >, \leq, \geq$, or $\neq$.

solution of an inequality in one variable (solución de una desigualdad en una variable) A value or values that make the inequality true.

What It Means For You

You can use inequalities to represent limits on the values in a situation so that the solutions make sense in a real-world context.

EXAMPLE

Anyone riding the large water slide at a park must be at least 40 inches tall.

Let h represent the heights that are allowed.

Height is at least 40 inches.

$$h \qquad \geq \qquad 40$$

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Solve linear … inequalities in one variable, …

Key Vocabulary

linear inequality in one variable (desigualdad lineal en una variable) An inequality that can be written in one of the following forms: $ax < b$, $ax > b$, $ax \leq b$, $ax \geq b$, or $ax \neq b$, where a and b are constants and $a \neq 0$.

What It Means For You

Solving inequalities lets you answer questions where a range of solutions is possible.

EXAMPLE

Solve the inequality for t to find what grades on the final exam will give Cleo a course grade of "A".

$705 + 2t \geq 895$ *Cleo has 705 points and needs at least 895.*

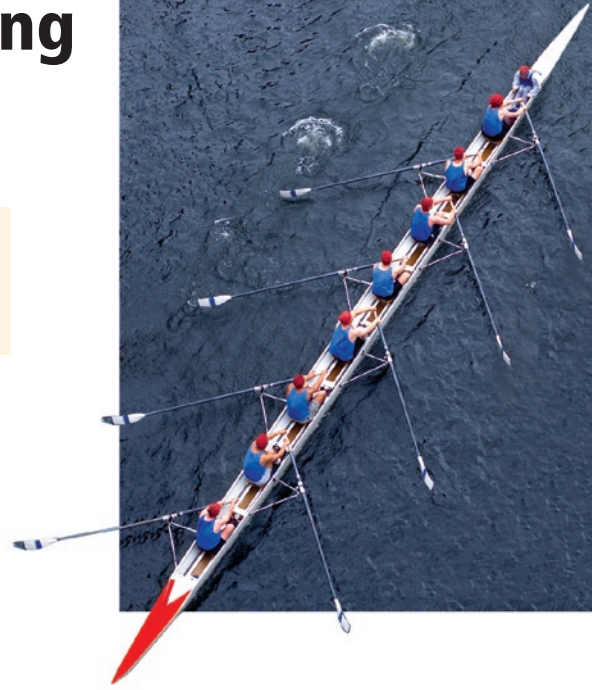$2t \geq 190$ *Subtract 705 from both sides.*

$t \geq 95$ *Divide both sides by 2.*

Cleo needs to earn a 95 or above on the final exam.

Graphing and Writing Inequalities

Essential Question: How can you graph and write inequalities?

Objectives
Identify solutions of inequalities in one variable.

Write and graph inequalities in one variable.

Vocabulary
inequality
solution of an inequality

Who uses this?
Members of a crew team can use inequalities to be sure they fall within a range of weights. (See Example 4.)

The athletes on a lightweight crew team must weigh 165 pounds or less. The acceptable weights for these athletes can be described using an *inequality*.

An **inequality** is a statement that two quantities are not equal. The quantities are compared by using one of the following signs:

Animated Math

$<$	$>$	$\leq$	$\geq$	$\neq$
$A < B$	$A > B$	$A \leq B$	$A \geq B$	$A \neq B$
A **is less than** *B*.	*A* **is greater than** *B*.	*A* **is less than or equal to** *B*.	*A* **is greater than or equal to** *B*.	*A* **is not equal to** *B*.

A **solution of an inequality** is any value of the variable that makes the inequality true.

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EXAMPLE 1

Identifying Solutions of Inequalities

Describe the solutions of $3 + x < 9$ in words.

Test values of *x* that are positive, negative, and 0.

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x	−2.75	0	5.99	6	6.01	6.1
3 + x	0.25	3	8.99	9	9.01	9.1
3 + x $\overset{?}{<}$ 9	0.25 $\overset{?}{<}$ 9	3 $\overset{?}{<}$ 9	8.99 $\overset{?}{<}$ 9	9 $\overset{?}{<}$ 9	9.01 $\overset{?}{<}$ 9	9.1 $\overset{?}{<}$ 9
Solution?	Yes	Yes	Yes	No	No	No

When the value of x is a number less than 6, the value of 3 + x is less than 9.
When the value of x is 6, the value of 3 + x is equal to 9.
When the value of x is a number greater than 6, the value of 3 + x is greater than 9.

The solutions of $3 + x < 9$ are numbers less than 6.

Writing Math

The solutions in Example 1 can be written in *set-builder notation* as $\{x \mid x < 6\}$, read as "*x* such that *x* is less than 6."

CHECK IT OUT!

1. Describe the solutions of $2p > 8$ in words.

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An inequality like $3 + x < 9$ has too many solutions to list. You can use a graph on a number line to show all the solutions.

The solutions are shaded and an arrow shows that the solutions continue past those shown on the graph. To show that an endpoint is a solution, draw a solid circle at the number. To show that an endpoint is not a solution, draw an empty circle.
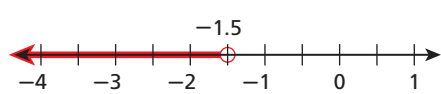
Know it! Note

Graphing Inequalities

WORDS	ALGEBRA	GRAPH
All real numbers less than 5	$x < 5$	
All real numbers greater than -1	$x > -1$	
All real numbers less than or equal to $\frac{1}{2}$	$x \leq \frac{1}{2}$	
All real numbers greater than or equal to 0	$x \geq 0$	

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EXAMPLE 2
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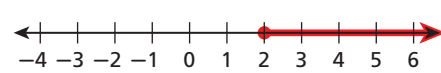
Graphing Inequalities

Graph each inequality.

A $b < -1.5$

Draw an empty circle at -1.5.
Shade all the numbers less than -1.5 and draw an arrow pointing to the left.

B $r \geq 2$

Draw a solid circle at 2.
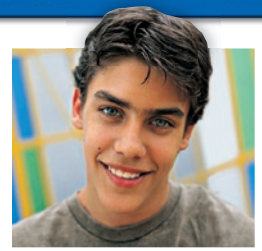*Shade all the numbers greater than 2 and draw an arrow pointing to the right.*

CHECK IT OUT!
Graph each inequality.
2a. $c > 2.5$ **2b.** $2^2 - 4 \geq w$ **2c.** $m \leq -3$

Student to Student

Graphing Inequalities

To know which direction to shade a graph, I write inequalities with the variable on the left side of the inequality symbol. I know that the symbol has to point to the same number after I rewrite the inequality.

For example, I write $4 < y$ as $y > 4$.

Now the inequality symbol points in the direction that I should draw the shaded arrow on my graph.
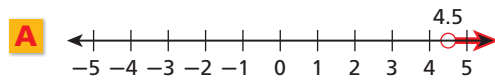

Victor Solomos
Palmer High School

EXAMPLE 3
Prep. for MCC9-12.A.REI.12
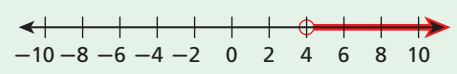
Writing an Inequality from a Graph

Write the inequality shown by each graph.

A

4.5

−5 −4 −3 −2 −1 0 1 2 3 4 5
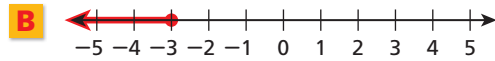
Use any variable. The arrow points to the right, so use either > or ≥.
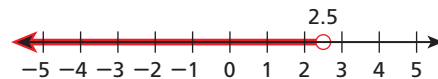*The empty circle at 4.5 means that 4.5 is not a solution, so use >.*

$h > 4.5$

B

−5 −4 −3 −2 −1 0 1 2 3 4 5

Use any variable. The arrow points to the left, so use either < or ≤.
The solid circle at −3 means that −3 is a solution, so use ≤.

$m \leq -3$

CHECK IT OUT!

3. Write the inequality shown by the graph.

2.5

−5 −4 −3 −2 −1 0 1 2 3 4 5
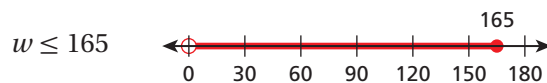

EXAMPLE 4
MCC9-12.A.CED.3

Sports Application

The members of a lightweight crew team can weigh no more than 165 pounds each. Define a variable and write an inequality for the acceptable weights of the team members. Graph the solutions.

Let w represent the weights that are allowed.

Athletes may weigh	no more than	165 pounds.
w	$\leq$	165

$w \leq 165$

165

0   30   60   90   120 150 180

Stop the graph at 0 because a person's weight must be a positive number.

Reading Math

"No more than" means "less than or equal to."

"At least" means "greater than or equal to."

CHECK IT OUT!

4. A store's employees earn at least $8.25 per hour. Define a variable and write an inequality for the amount the employees may earn per hour. Graph the solutions.

MCC.MP.6

MATHEMATICAL PRACTICES

Know it! Note

THINK AND DISCUSS

1. Compare the solutions of $x > 2$ and $x \geq 2$.

2. GET ORGANIZED Copy and complete the graphic organizer. Draw a graph in the first row and write the correct inequality in the second row.

Inequality	Graph
$x > 1$	
	−5 −4 −3 −2 −1 0 1

GUIDED PRACTICE

1. Vocabulary How is a *solution of an inequality* like a solution of an equation?

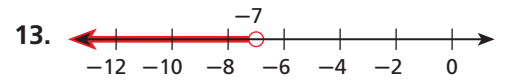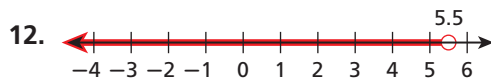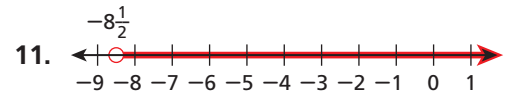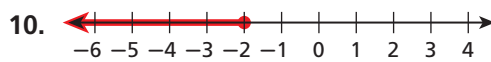SEE EXAMPLE 1 **Describe the solutions of each inequality in words.**

2. $g - 5 \geq 6$ **3.** $-2 < h + 1$ **4.** $20 > 5t$ **5.** $5 - x \leq 2$

SEE EXAMPLE 2 **Graph each inequality.**

6. $x < -5$ **7.** $c \geq 3\frac{1}{2}$ **8.** $(4 - 2)^3 > m$ **9.** $p \geq \sqrt{17 + 8}$

SEE EXAMPLE 3 **Write the inequality shown by each graph.**

10.

11.

12.

13.

14.

15.
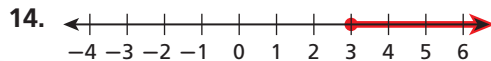
SEE EXAMPLE 4 **Define a variable and write an inequality for each situation. Graph the solutions.**

16. There must be at least 20 club members present in order to hold a meeting.

17. A trainer advises an athlete to keep his heart rate under 140 beats per minute.

PRACTICE AND PROBLEM SOLVING

Independent Practice

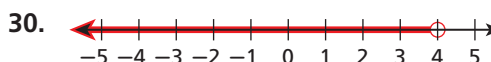| For Exercises	See Example
18–21	1
22–25	2
26–31	3
32–33	4

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Online Extra Practice

Describe the solutions of each inequality in words.
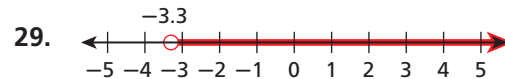
18. $-2t > -8$ **19.** $0 > w - 2$ **20.** $3k > 9$ **21.** $\frac{1}{2}b \leq 6$

Graph each inequality.

22. $7 < x$ **23.** $t \leq -\frac{1}{2}$ **24.** $d > 4(5 - 8)$ **25.** $t \leq 3^2 - 2^2$

Write the inequality shown by each graph.

26.

27.

28.

29.

30.

31.
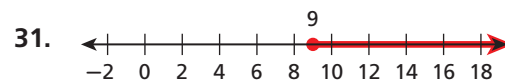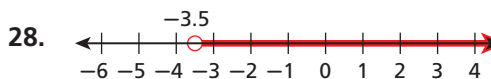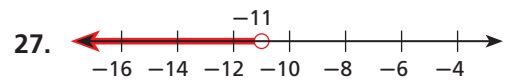
Define a variable and write an inequality for each situation. Graph the solutions.

32. The maximum speed allowed on Main Street is 25 miles per hour.

33. Applicants must have at least 5 years of experience.

Write each inequality in words.

34. $x > 7$ **35.** $h < -5$ **36.** $d \leq 23$ **37.** $r \geq -2$

Write each inequality with the variable on the left. Graph the solutions.

38. $19 < g$ **39.** $17 \geq p$ **40.** $10 < e$ **41.** $0 < f$

Define a variable and write an inequality for each situation. Graph the solutions.

42. The highest temperature ever recorded on Earth was 135.9 °F at Al Aziziyah, Libya, on September 13, 1922.

43. Businesses with profits less than $10,000 per year will be shut down.

44. You must be at least 46 inches tall to ride a roller coaster at an amusement park.

45. Due to a medical condition, a hiker can hike only in areas with an elevation no more than 5000 feet above sea level.

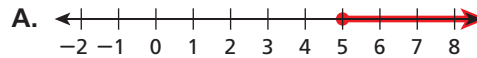**Write a real-world situation that could be described by each inequality.**

46. $x \geq 0$ **47.** $x < 10$ **48.** $x \leq 12$ **49.** $x > 8.5$

Match each inequality with its graph.

50. $x \geq 5$

A.

51. $x < 5$

B.

52. $x > 5$

C.

53. $x \leq 5$

D.
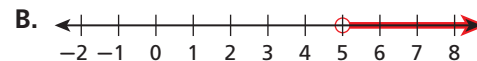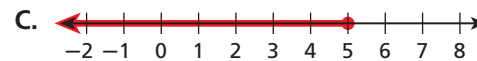
H.O.T. 54. /// **ERROR ANALYSIS** /// Two students graphed the inequality $4 > b$. Which graph is incorrect? Explain the error.

Ⓐ

Ⓑ
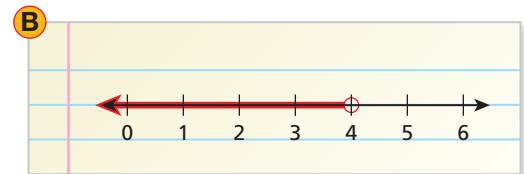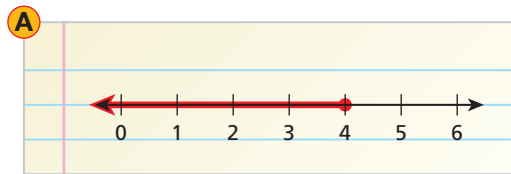
Real-World Connections

55. a. Mirna earned $125 baby-sitting during the spring break. She needs to save $90 for the German Club trip. She wants to spend the remainder of the money shopping. Write an inequality to show how much she can spend.

b. Graph the inequality you wrote in part **a.**

c. Mirna spends $15 on a bracelet. Write an inequality to show how much money she has left to spend.

56. Critical Thinking Graph all positive integer solutions of the inequality $x < 5$.

H.O.T. 57. Write About It Explain how to write an inequality that is modeled by a graph. What characteristics do you look for in the graph?

58. Write About It You were told in the lesson that the phrase "no more than" means "less than or equal to" and the phrase "at least" means "greater than or equal to."
 a. What does the phrase "at most" mean?
 b. What does the phrase "no less than" mean?

TEST PREP

59. Which is NOT a solution of the inequality $5 - 2x \geq -3$?
 (A) 0   (B) 2   (C) 4   (D) 5

60. Which is NOT a solution of the inequality $3 - x < 2$?
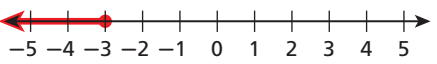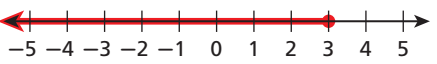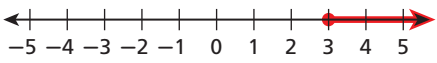 (F) 1   (G) 2   (H) 3   (J) 4

61. Which graph represents the solutions of $-2 \leq 1 - t$?

CHALLENGE AND EXTEND

Describe the values for x and y that make each inequality true.

62. $x + y \leq |x + y|$ **63.** $x^2 < xy$ **64.** $x - y \geq y - x$

Complete each statement. Write $<$ or $>$.

65. If $a > b$, then b ___ a.   **66.** If $x > y$ and $y > z$, then x ___ z.

67. Name a value of x that makes the statement $0.35 < x < 1.27$ true.

68. Is $\frac{5}{6}$ a solution of $x < 1$? How many solutions of $x < 1$ are between 0 and 1?

69. Write About It Explain how to graph all the solutions of $x \neq 5$.

FOCUS ON MATHEMATICAL PRACTICES

H.O.T. 70. Modeling In order for Ramon to remain in his current weight class for a wrestling match on Saturday morning, he must weigh in at 152 pounds or more, but less than 160 pounds. Write a pair of inequalities that expresses the set of acceptable weights for Ramon. Define your variable.

H.O.T. 71. Problem Solving Cary is making brownies using a recipe that calls for "at least 5 cups of flour but no more than 6 cups of flour." The only measuring cup he could find holds one quarter of a cup. Write a pair of inequalities to express how many *quarter cups* of flour Cary can use.

H.O.T. 72. Analysis Imani and Trey are planning the seating at their wedding reception. They have 168 guests and each table can hold up to 16 guests, so they calculate that they need at least 10.5 tables to seat all of their guests. Graph their solution. In this context, how is the graph inaccurate? Make another graph that takes the context into account.

Solving Inequalities by Adding or Subtracting

? **_Essential Question:_** How can you use addition or subtraction to solve inequalities?

Objectives
Solve one-step inequalities by using addition.

Solve one-step inequalities by using subtraction.

Who uses this?
You can use inequalities to determine how many more photos you can take. (See Example 2.)

Tenea has a cell phone that also takes pictures. After taking some photos, Tenea can use a one-step inequality to determine how many more photos she can take.

Solving one-step inequalities is much like solving one-step equations. To solve an inequality, you need to isolate the variable using the properties of inequality and inverse operations.

Know it! Note

Properties of Inequality

Addition and Subtraction

WORDS	NUMBERS	ALGEBRA
Addition You can add the same number to both sides of an inequality, and the statement will still be true.	$3 < 8$ $3 + 2 < 8 + 2$ $5 < 10$	$a < b$ $a + c < b + c$
Subtraction You can subtract the same number from both sides of an inequality, and the statement will still be true.	$9 < 12$ $9 - 5 < 12 - 5$ $4 < 7$	$a < b$ $a - c < b - c$

These properties are also true for inequalities that use the symbols $>$, $\geq$, and $\leq$.

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EXAMPLE 1 Using Addition and Subtraction to Solve Inequalities

Solve each inequality and graph the solutions.
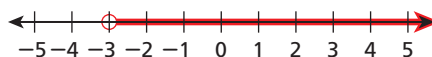
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A $x + 9 < 15$

$$x + 9 < 15$$
$$\underline{-9 \quad -9}$$
$$x \quad < \quad 6$$

Since 9 is added to x, subtract 9 from both sides to undo the addition.
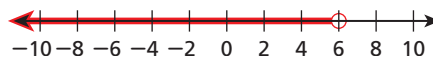
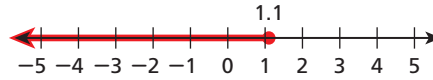**B** $d - 3 > -6$

$$d - 3 > -6$$
$$\underline{+3 \quad +3}$$
$$d \quad > -3$$

Since 3 is subtracted from d, add 3 to both sides to undo the subtraction.

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Solve each inequality and graph the solutions.

C $0.7 \geq n - 0.4$

$0.7 \geq n - 0.4$ *Since 0.4 is subtracted from n, add 0.4 to both sides to undo the subtraction.*

$\underline{+\,0.4 \qquad +\,0.4}$

$1.1 \geq n$

$n \leq 1.1$

CHECK IT OUT! Solve each inequality and graph the solutions.

1a. $s + 1 \leq 10$ **1b.** $2\frac{1}{2} > -3 + t$ **1c.** $q - 3.5 < 7.5$

Since there can be an infinite number of solutions to an inequality, it is not possible to check all the solutions. You can check the endpoint and the direction of the inequality symbol.

The solutions of $x + 9 < 15$ are given by $x < 6$.

Step 1 Check the endpoint.

Substitute 6 for x in the related equation $x + 9 = 15$. The endpoint should be a solution of the equation.

$x + 9 = 15$

$6 + 9$	15
15	15 ✓

Step 2 Check the inequality symbol.

Substitute a number less than 6 for x in the original inequality. The number you choose should be a solution of the inequality.

$x + 9 < 15$

$4 + 9$	$<$	15
13	$<$	15 ✓

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MCC9-12.N.Q.1

EXAMPLE 2

Problem Solving Application

The memory in Tenea's camera phone allows her to take up to 20 pictures. Tenea has already taken 16 pictures. Write, solve, and graph an inequality to show how many more pictures Tenea could take.

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1 Understand the Problem

The **answer** will be an inequality and a graph that show all the possible numbers of pictures that Tenea can take.

List the important information:
- Tenea can take up to, or *at most*, 20 pictures.
- Tenea has taken 16 pictures already.

MATHEMATICAL PRACTICES

Make sence of problems and persevere in solving them.

2 Make a Plan

Write an inequality.
Let p represent the remaining number of pictures Tenea can take.

Number taken	plus	number remaining	is at most	20 pictures.
16	+	p	$\leq$	20

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3 Solve

$$16 + p \leq 20$$

Since 16 is added to p, subtract 16 from both sides to undo the addition.

$$\underline{-16 \qquad -16}$$

$$p \leq 4$$

It is not reasonable for Tenea to take a negative or fractional number of pictures, so graph the nonnegative integers less than or equal to 4.

Tenea could take 0, 1, 2, 3, or 4 more pictures.

4 Look Back

Check Check the endpoint, 4.

$$16 + \mathbf{p} = 20$$

$16 + \mathbf{4}$	20
20	20 ✓

Check a number less than 4.

$$16 + \mathbf{p} \leq 20$$

$16 + \mathbf{2}$	$\leq$	20
18	$\leq$	20 ✓

Adding 0, 1, 2, 3, or 4 more pictures will not exceed 20.

CHECK IT OUT!

2. The Recommended Dietary Allowance (RDA) of iron for a female in Sarah's age group (14–18 years) is 15 mg per day. Sarah has consumed 11 mg of iron today. Write and solve an inequality to show how many more milligrams of iron Sarah can consume without exceeding the RDA.

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EXAMPLE 3

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Sports Application

Josh can bench press 220 pounds. He wants to bench press at least 250 pounds. Write and solve an inequality to determine how many more pounds Josh must lift to reach his goal. Check your answer.

Let p represent the number of additional pounds Josh must lift.

220 pounds	plus	additional pounds	is at least	250 pounds.
220	+	p	$\geq$	250

$$220 + p \geq 250$$

Since 220 is added to p, subtract 220 from both sides to undo the addition.

$$\underline{-220 \qquad -220}$$

$$p \geq 30$$

Check Check the endpoint, 30.

$$220 + \mathbf{p} = 250$$

$220 + \mathbf{30}$	250
250	250 ✓

Check a number greater than 30.

$$220 + \mathbf{p} \geq 250$$

| $220 + \mathbf{40}$	$\geq$	250
260	$\geq$	250 ✓

Josh must lift at least 30 additional pounds to reach his goal.

CHECK IT OUT!

3. **What if...?** Josh has reached his goal of 250 pounds and now wants to try to break the school record of 282 pounds. Write and solve an inequality to determine how many more pounds Josh needs to break the school record. Check your answer.

THINK AND DISCUSS

1. Show how to check your solution to Example 1B.

2. Explain how the Addition and Subtraction Properties of Inequality are like the Addition and Subtraction Properties of Equality.

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an inequality that you must use the specified property to solve. Then solve and graph the inequality.

Properties of Inequality
— Addition — Subtraction
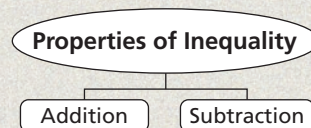
4-2 Exercises

GUIDED PRACTICE

SEE EXAMPLE 1 Solve each inequality and graph the solutions.

1. $12 < p + 6$ **2.** $w + 3 \geq 4$ **3.** $-5 + x \leq -20$ **4.** $z - 2 > -11$

SEE EXAMPLE 2 **5. Health** For adults, the maximum safe water temperature in a spa is 104 °F. The water temperature in Bill's spa is 102 °F. The temperature is increased by t °F. Write, solve, and graph an inequality to show the values of t for which the water temperature is still safe.

SEE EXAMPLE 3 **6. Consumer Economics** A local restaurant will deliver food to your house if the purchase amount of your order is at least $25.00. The total for part of your order is $17.95. Write and solve an inequality to determine how much more you must spend for the restaurant to deliver your order.

PRACTICE AND PROBLEM SOLVING

Independent Practice	
For Exercises	See Example
7–10	1
11	2
12	3

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Online Extra Practice

Solve each inequality and graph the solutions.

7. $a - 3 \geq 2$ **8.** $2.5 > q - 0.8$ **9.** $-45 + x < -30$ **10.** $r + \frac{1}{4} \leq \frac{3}{4}$

11. Engineering The maximum load for a certain elevator is 2000 pounds. The total weight of the passengers on the elevator is 1400 pounds. A delivery man who weighs 243 pounds enters the elevator with a crate of weight w. Write, solve, and graph an inequality to show the values of w that will not exceed the weight limit of the elevator.

12. Transportation The gas tank in Mindy's car holds at most 15 gallons. She has already filled the tank with 7 gallons of gas. She will continue to fill the tank with g gallons more. Write and solve an inequality that shows all values of g that Mindy can add to the car's tank.

Write an inequality to represent each statement. Solve the inequality and graph the solutions.

13. Ten less than a number x is greater than 32.

14. A number n increased by 6 is less than or equal to 4.

15. A number r decreased by 13 is at most 15.

Solve each inequality and graph the solutions.

16. $x + 4 \leq 2$ **17.** $-12 + q > 39$ **18.** $x + \dfrac{3}{5} < 7$ **19.** $4.8 \geq p + 4$

20. $-12 \leq x - 12$ **21.** $4 < 206 + c$ **22.** $y - \dfrac{1}{3} > \dfrac{2}{3}$ **23.** $x + 1.4 \geq 1.4$

24. Use the inequality $s + 12 \geq 20$ to fill in the missing numbers.

    a. $s \geq$ ▦ **b.** $s +$ ▦ ≥ 30 **c.** $s - 8 \geq$ ▦

25. **Health** A particular type of contact lens can be worn up to 30 days in a row. Alex has been wearing these contact lenses for 21 days. Write, solve, and graph an inequality to show how many more days Alex could wear his contact lenses.

Solve each inequality and match the solutions to the correct graph.

26. $1 \leq x - 2$

A.
```
←|—+—+—+—+—+—+—+—+—+—⊕—+—+→
 −5 −4 −3 −2 −1  0  1  2  3  4  5
```

27. $8 > x - (-5)$

B.
```
←——————————————————⊕—+—+→
 −5 −4 −3 −2 −1  0  1  2  3  4  5
```

28. $x + 6 > 9$

C.
```
←|—+—+—+—+—+—+—+—●————————→
 −5 −4 −3 −2 −1  0  1  2  3  4  5
```

29. $-4 \geq x - 7$

D.
```
←————————————————————●—+—+→
 −5 −4 −3 −2 −1  0  1  2  3  4  5
```

H.O.T. 30. **Estimation** Is $x < 10$ a reasonable estimate for the solutions to the inequality $11.879 + x < 21.709$? Explain your answer.

31. **Sports** At the Seattle Mariners baseball team's home games, there are 45,611 seats in the four areas listed in the table. Suppose all the suite level and club level seats during a game are filled. Write and solve an inequality to determine how many people p could be sitting in the other types of seats.

Mariners Home Game Seating	
Type of Seat	**Number of Seats**
Main bowl	24,399
Upper bowl	16,022
Club level	4,254
Suite level	936

32. **Critical Thinking** Recall that a balance scale was used to model solving equations. Describe how a balance scale could model solving inequalities.

33. **Critical Thinking** Explain why $x + 4 \geq 6$ and $x - 4 \geq -2$ have the same solutions.
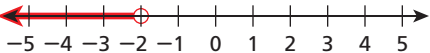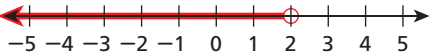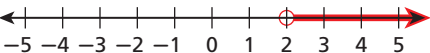
H.O.T. 34. **Write About It** How do the solutions of $x + 2 \geq 3$ differ from the solutions of $x + 2 > 3$? How do the graphs of the solutions differ?

35. a. Daryl finds that the distance from Columbus, Ohio, to Washington, D.C., is 411 miles. What is the round-trip distance?

    b. Daryl can afford to drive a total of 1000 miles. Write an inequality to show the number of miles m he can drive while in Washington, D.C.

    c. Solve the inequality and graph the solutions on a number line. Show that your answer is reasonable.

36. Which is a reasonable solution of $4.7367 + p < 20.1784$?

　Ⓐ 15　　　　　Ⓑ 16　　　　　Ⓒ 24　　　　　Ⓓ 25

37. Which statement can be modeled by $x + 3 \leq 12$?

　Ⓕ Sam has 3 bottles of water. Together, Sam and Dave have at most 12 bottles of water.

　Ⓖ Jennie sold 3 cookbooks. To earn a prize, Jennie must sell at least 12 cookbooks.

　Ⓗ Peter has 3 baseball hats. Peter and his brothers have fewer than 12 baseball hats.

　Ⓙ Kathy swam 3 laps in the pool this week. She must swim more than 12 laps.

38. Which graph represents the solutions of $p + 3 < 1$?

Ⓐ 　　Ⓒ

Ⓑ 　　Ⓓ

39. Which inequality does NOT have the same solutions as $n + 12 \leq 26$?

　Ⓕ $n \leq 14$　　　Ⓖ $n + 6 \leq 20$　　　Ⓗ $10 \geq n - 4$　　　Ⓙ $n - 12 \leq 14$

CHALLENGE AND EXTEND

Solve each inequality and graph the solutions.

40. $6\dfrac{9}{10} \geq 4\dfrac{4}{5} + x$　　　　**41.** $r - 1\dfrac{2}{5} \leq 3\dfrac{7}{10}$　　　　**42.** $6\dfrac{2}{3} + m > 7\dfrac{1}{6}$

Determine whether each statement is sometimes, always, or never true. Explain.

43. $a + b > a - b$

44. If $a > c$, then $a + b > c + b$.

45. If $a > b$ and $c > d$, then $a + c > b + d$.

46. If $x + b > c$ and $x > 0$ have the same solutions, what is the relationship between b and c?

FOCUS ON MATHEMATICAL PRACTICES

MATHEMATICAL PRACTICES

H.O.T. 47. Estimation In 10 weeks, Yuri wants to have enough money to buy a racing bicycle that costs $1487.95. He currently has $292.50 in his savings account.

　a. Write an inequality that expresses how much Yuri still needs to save.

　b. Rewrite this inequality in a simpler form by rounding all numerical values to the nearest $100, and then solve it.

　c. Use the solution you just found to estimate how much money Yuri needs to save each week in order to purchase the bicycle.

H.O.T. 48. Modeling A psychotherapist needs to complete a minimum number of internship hours before he or she can receive certification. Candace wrote the inequality $1840 + h \geq 3000$ to represent the hours she needs complete her certification. In total, how many hours does Candace need to serve as an intern? How many hours has Candace completed? How many hours does Candace still need to perform?

Solving Inequalities by Multiplying or Dividing

? **Essential Question:** How can you use multiplication or division to solve inequalities?

Objectives
Solve one-step inequalities by using multiplication.

Solve one-step inequalities by using division.

Who uses this?
You can solve an inequality to determine how much you can buy with a certain amount of money. (See Example 3.)

Remember, solving inequalities is similar to solving equations. To solve an inequality that contains multiplication or division, undo the operation by dividing or multiplying both sides of the inequality by the same number.

The rules below show the properties of inequality for multiplying or dividing by a positive number. The rules for multiplying or dividing by a negative number appear later in this lesson.

"This is all I have, so I'll take 3 pencils, 3 notebooks, a binder, and 0.9 calculators."

Jonny Hawkins/CartoonResource.com

Know it! **Note**

Properties of Inequality

Multiplication and Division by Positive Numbers

WORDS	NUMBERS	ALGEBRA
Multiplication You can multiply both sides of an inequality by the same *positive* number, and the statement will still be true.	$7 < 12$ $7(3) < 12(3)$ $21 < 36$	If $a < b$ and $c > 0$, then $ac < bc$.
Division You can divide both sides of an inequality by the same *positive* number, and the statement will still be true.	$15 < 35$ $\dfrac{15}{5} < \dfrac{35}{5}$ $3 < 7$	If $a < b$ and $c > 0$, then $\dfrac{a}{c} < \dfrac{b}{c}$.

These properties are also true for inequalities that use the symbols $>$, $\geq$, and $\leq$.

COMMON CORE GPS MCC9-12.A.REI.3

EXAMPLE 1 Multiplying or Dividing by a Positive Number

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Solve each inequality and graph the solutions.

A $3x > -27$

$3x > -27$ *Since x is multiplied by 3, divide both sides by 3 to undo the multiplication.*

$\dfrac{3x}{3} > \dfrac{-27}{3}$

$x > -9$

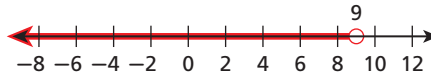**Solve each inequality and graph the solutions.**

B $\frac{2}{3}r < 6$

$\frac{2}{3}r < 6$ *Since r is multiplied by $\frac{2}{3}$, multiply both sides by the reciprocal of $\frac{2}{3}$.*

$\frac{3}{2}\left(\frac{2}{3}r\right) < \frac{3}{2}(6)$

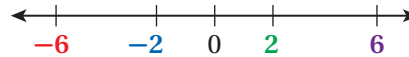$r < 9$

Solve each inequality and graph the solutions.

1a. $4k > 24$ **1b.** $-50 \geq 5q$ **1c.** $\frac{3}{4}g > 27$

What happens when you multiply or divide both sides of an inequality by a negative number?

Look at the number line below.

$2 < 6$ $6 > -2$

$-2 \;\blacksquare\; -6$ *Multiply both sides by -1.* $-6 \;\blacksquare\; 2$ *Multiply both sides by -1.*

$-2 > -6$ *Use the number line to determine the direction of the inequality.* $-6 < 2$ *Use the number line to determine the direction of the inequality.*

Notice that when you multiply (or divide) both sides of an inequality by a negative number, you must reverse the inequality symbol. This means there is another set of properties of inequality for multiplying or dividing by a negative number.

Know it! Note

Properties of Inequality

Multiplication and Division by Negative Numbers

WORDS	NUMBERS	ALGEBRA
Multiplication If you multiply both sides of an inequality by the same *negative* number, you must reverse the inequality symbol for the statement to still be true.	$8 > 4$ $8(-2) < 4(-2)$ $-16 < -8$	If $a > b$ and $c < 0$, then $ac < bc$.
Division If you divide both sides of an inequality by the same *negative* number, you must reverse the inequality symbol for the statement to still be true.	$12 > 4$ $\dfrac{12}{-4} < \dfrac{4}{-4}$ $-3 < -1$	If $a > b$ and $c < 0$, then $\dfrac{a}{c} < \dfrac{b}{c}$.
These properties are also true for inequalities that use the symbols $<$, $\geq$, and $\leq$.		

EXAMPLE 2

Multiplying or Dividing by a Negative Number

Solve each inequality and graph the solutions.

A $-8x > 72$

$$\frac{-8x}{-8} < \frac{72}{-8}$$

Since x is multiplied by −8, divide both sides by −8.
Change > to <.

$$x < -9$$
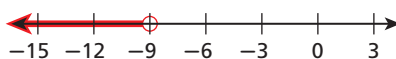
B $-3 \le \dfrac{x}{-5}$

$$-5(-3) \ge -5\left(\frac{x}{-5}\right)$$

Since x is divided by −5, multiply both sides by −5.
Change ≤ to ≥.

$$15 \ge x \ (\text{or } x \le 15)$$
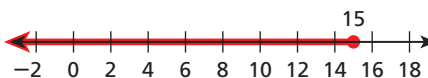
CHECK IT OUT! Solve each inequality and graph the solutions.

2a. $10 \ge -x$ **2b.** $4.25 > -0.25h$

EXAMPLE 3

Consumer Application

Ryan has a $16 gift card for a health store where a smoothie costs $2.50 with tax. What are the possible numbers of smoothies that Ryan can buy?

Let *s* represent the number of smoothies Ryan can buy.

$2.50	times	number of smoothies	is at most	$16.00.
2.50	•	*s*	≤	16.00

$$2.50s \le 16.00$$

$$\frac{2.50s}{2.50} \le \frac{16.00}{2.50}$$

Since s is multiplied by 2.50, divide both sides by 2.50.
The symbol does not change.

$$s \le 6.4$$

Ryan can buy only a whole number of smoothies.

Ryan can buy 0, 1, 2, 3, 4, 5, or 6 smoothies.

CHECK IT OUT!

3. A pitcher holds 128 ounces of juice. What are the possible numbers of 10-ounce servings that one pitcher can fill?

MCC.MP.8   **MATHEMATICAL PRACTICES**

THINK AND DISCUSS

1. Compare the Multiplication and Division Properties of Inequality and the Multiplication and Division Properties of Equality.

2. GET ORGANIZED Copy and complete the graphic organizer. In each cell, write and solve an inequality.

Solving Inequalities by Using Multiplication and Division		
	By a Positive Number	**By a Negative Number**
Divide		
Multiply		

GUIDED PRACTICE

Solve each inequality and graph the solutions.

SEE EXAMPLE **1**

1. $3b > 27$ **2.** $-40 \geq 8b$ **3.** $\dfrac{d}{3} > 6$ **4.** $24d \leq 6$

5. $1.1m \leq 1.21$ **6.** $\dfrac{2}{3}k > 6$ **7.** $9s > -18$ **8.** $\dfrac{4}{5} \geq \dfrac{r}{2}$

SEE EXAMPLE **2**

9. $-2x < -10$ **10.** $\dfrac{b}{-2} \geq 8$ **11.** $-3.5n < 1.4$ **12.** $4 > -8g$

13. $\dfrac{d}{-6} < \dfrac{1}{2}$ **14.** $-10h \geq -6$ **15.** $12 > \dfrac{t}{-6}$ **16.** $-\dfrac{1}{2}m \geq -7$

SEE EXAMPLE **3**

17. Travel Tom saved $550 to go on a school trip. The cost for a hotel room, including tax, is $80 per night. What are the possible numbers of nights Tom can stay at the hotel?

PRACTICE AND PROBLEM SOLVING

For Exercises	See Example
18–29	1
30–41	2
42	3

Independent Practice

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Solve each inequality and graph the solutions.

18. $10 < 2t$ **19.** $\dfrac{1}{3}j \leq 4$ **20.** $-80 < 8c$ **21.** $21 > 3d$

22. $\dfrac{w}{4} \geq -2$ **23.** $\dfrac{h}{4} \leq \dfrac{2}{7}$ **24.** $6y < 4.2$ **25.** $12c \leq -144$

26. $\dfrac{4}{5}x \geq \dfrac{2}{5}$ **27.** $6b \geq \dfrac{3}{5}$ **28.** $-25 > 10p$ **29.** $\dfrac{b}{8} \leq -2$

30. $-9a > 81$ **31.** $\dfrac{1}{2} < \dfrac{r}{-3}$ **32.** $-6p > 0.6$ **33.** $\dfrac{y}{-4} > -\dfrac{1}{2}$

34. $-\dfrac{1}{6}f < 5$ **35.** $-2.25t < -9$ **36.** $24 \leq -10w$ **37.** $-11z > 121$

38. $\dfrac{3}{5} < \dfrac{f}{-5}$ **39.** $-k \geq 7$ **40.** $-2.2b < -7.7$ **41.** $16 \geq -\dfrac{4}{3}p$

42. Camping The rope Roz brought with her camping gear is 54 inches long. Roz needs to cut shorter pieces of rope that are each 18 inches long. What are the possible number of pieces Roz can cut?

Solve each inequality and graph the solutions.

43. $-8x < 24$ **44.** $3t \leq 24$ **45.** $\dfrac{1}{4}x < 5$ **46.** $\dfrac{4}{5}p \geq -24$

47. $54 \leq -9p$ **48.** $3t > -\dfrac{1}{2}$ **49.** $-\dfrac{3}{4}b > -\dfrac{3}{2}$ **50.** $216 > 3.6r$

Write an inequality for each statement. Solve the inequality and graph the solutions.

51. The product of a number and 7 is not less than 21.

52. The quotient of h and -6 is at least 5.

53. The product of $-\dfrac{4}{5}$ and b is at most -16.

54. Ten is no more than the quotient of t and 4.

H.O.T. 55. Write About It Explain how you know whether to reverse the inequality symbol when solving an inequality.

56. Geometry The area of a rectangle is at most 21 square inches. The width of the rectangle is 3.5 inches. What are the possible measurements for the length of the rectangle?

Solve each inequality and match the solution to the correct graph.

57. $-0.5t \geq 1.5$
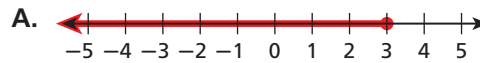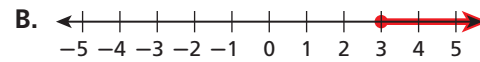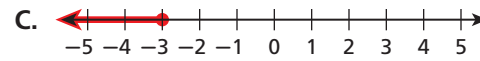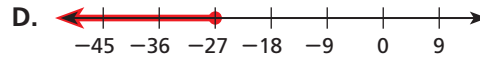
A.

$-5\ -4\ -3\ -2\ -1\ \ 0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5$

58. $\dfrac{1}{9}t \leq -3$

B.

$-5\ -4\ -3\ -2\ -1\ \ 0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5$

59. $-13.5 \leq -4.5t$

C.

$-5\ -4\ -3\ -2\ -1\ \ 0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5$

60. $\dfrac{t}{-6} \leq -\dfrac{1}{2}$

D.

$-45\ -36\ -27\ -18\ \ -9\ \ \ 0\ \ \ 9$

61. Animals A wildlife shelter is home to birds, mammals, and reptiles. If cat chow is sold in 20 lb bags, what is the least number of bags of cat chow needed for one year at this shelter?

Food Consumed at a Wildlife Shelter per Week	
Type of Food	**Amount of Food (lb)**
Grapes	4
Mixed seed	10
Peanuts	5
Cat chow	10
Kitten chow	5

62. Education In order to earn an A in a college math class, a student must score no less than 90% of all possible points. One semester, a student with 567 points earned an A in the class. Write an inequality to show the numbers of points possible.

H.O.T. 63. Critical Thinking Explain why you cannot solve an inequality by multiplying both sides by zero.

H.O.T. 64. ///**ERROR ANALYSIS**/// Two students have different answers for a homework problem. Which answer is incorrect? Explain the error.

A

$9m \geq -27$

$\dfrac{9m}{9} \geq \dfrac{-27}{9}$

$m \leq -3$

B

$9m \geq -27$

$\dfrac{9m}{9} \geq \dfrac{-27}{9}$

$m \geq -3$

65. Jan has a budget of $800 for catering. The catering company charges $12.50 per guest. Write and solve an inequality to show the numbers of guests Jan can invite.

66. a. The Swimming Club can spend a total of $250 for hotel rooms for its spring trip. One hotel costs $75 per night. Write an inequality to find the number of rooms the club can reserve at this hotel. Let n be the number of rooms.

b. Solve the inequality you wrote in part **a.** Graph the solutions on a number line. Make sure your answer is reasonable.

c. Another hotel offers a rate of $65 per night. Does this allow the club to reserve more rooms? Explain your reasoning.

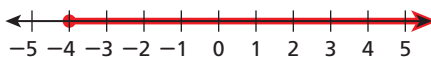**67.** Which inequality does NOT have the same solutions as $-\frac{2}{3}y > 4$?

Ⓐ $12 < -2y$

Ⓒ $-\frac{3}{4}y > \frac{9}{2}$

Ⓑ $\frac{y}{2} < -12$

Ⓓ $-3y > 18$

68. The solutions of which inequality are NOT represented by the following graph?

Ⓕ $\frac{x}{2} \geq -2$

Ⓗ $3x \geq -12$

Ⓖ $-5x \geq 20$

Ⓙ $-7x \leq 28$

69. Which inequality can be used to find the number of 39-cent stamps you can purchase for $4.00?

Ⓐ $0.39s \geq 4.00$

Ⓒ $\frac{s}{0.39} \leq 4.00$

Ⓑ $0.39s \leq 4.00$

Ⓓ $\frac{4.00}{0.39} \leq s$

70. Short Response Write three different inequalities that have the same solutions as $x > 4$. Show your work and explain each step.

CHALLENGE AND EXTEND

Solve each inequality.

71. $2\frac{1}{3} \leq -\frac{5}{6}g$ **72.** $\frac{2x}{3} < 8.25$ **73.** $2\frac{5}{8}m > \frac{7}{10}$ **74.** $3\frac{3}{5}f \geq 14\frac{2}{5}$

75. Estimation What is the greatest possible integer solution of the inequality $3.806x < 19.902$?

76. Critical Thinking The Transitive Property of Equality states that if $a = b$ and $b = c$, then $a = c$. Is there a Transitive Property of Inequality using the symbol $<$? Give an example to support your answer.

77. Critical Thinking The Symmetric Property of Equality states that if $a = b$, then $b = a$. Is there a Symmetric Property of Inequality? Give an example to support your answer.

MATHEMATICAL PRACTICES

FOCUS ON MATHEMATICAL PRACTICES

H.O.T. 78. Error Analysis Marigold solves an inequality as shown.

$$\frac{4}{5}x \geq -20$$
$$\frac{5}{4}\left(\frac{4}{5}x\right) \geq \frac{5}{4}(-20)$$
$$x \leq -\frac{100}{4}$$
$$x \leq -25$$

What mistake did Marigold make? What is the correct answer?

H.O.T. 79. Make a Conjecture The solution to the inequality $kx < 6$ is $x > -2$.

a. What can you say about the value of k just by looking at the inequality and its solution, without actually solving the problem?

b. Find another solution for x by dividing both sides of $kx < 6$ by k.

c. Use $x > -2$ and your solution from part b to make an equation, then solve for k. Does the solution support your conjecture from part a?

Ready to Go On?

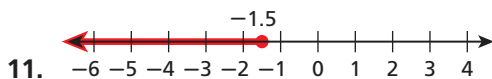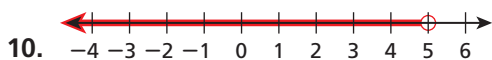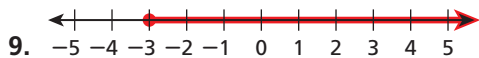## 4-1 Graphing and Writing Inequalities

Describe the solutions of each inequality in words.

1. $-2 < r$ **2.** $t - 1 \leq 7$ **3.** $2s \geq 6$ **4.** $4 > 5 - x$

Graph each inequality.

5. $x > -2$ **6.** $m \leq 1\frac{1}{2}$ **7.** $g < \sqrt{8} + 1$ **8.** $h \geq 2^3$

Write the inequality shown by each graph.

9.

$-5\ -4\ -3\ -2\ -1\ \ 0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5$

10.

$-4\ -3\ -2\ -1\ \ 0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6$

11.

-1.5
$-6\ -5\ -4\ -3\ -2\ -1\ \ 0\ \ 1\ \ 2\ \ 3\ \ 4$

Write an inequality for each situation and graph the solutions.

12. You must purchase at least 5 tickets to receive a discount.

13. Children under 13 are not admitted to certain movies without an adult.

14. A cell phone plan allows up to 250 free minutes per month.

4-2 Solving Inequalities by Adding or Subtracting

Solve each inequality and graph the solutions.

15. $k + 5 \leq 7$ **16.** $4 > p - 3$ **17.** $r - 8 \geq -12$ **18.** $-3 + p < -6$

19. Allie must sell at least 50 gift baskets for the band fund-raiser. She already sold 36 baskets. Write and solve an inequality to determine how many more baskets Allie must sell for the fund-raiser.

20. Dante has at most $12 to spend on entertainment each week. So far this week, he spent $7.50. Write and solve an inequality to determine how much money Dante can spend on entertainment the rest of the week.
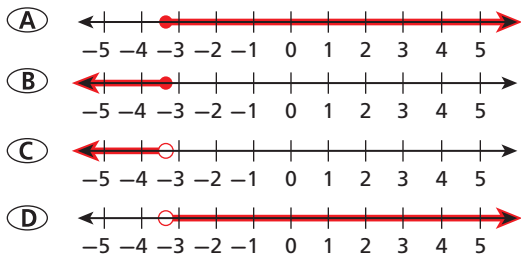
4-3 Solving Inequalities by Multiplying or Dividing

Solve each inequality and graph the solutions.

21. $-4x < 8$ **22.** $\frac{d}{3} \geq -3$ **23.** $\frac{3}{4}t \leq 12$ **24.** $8 > -16c$

25. A spool of ribbon is 80 inches long. Riley needs to cut strips of ribbon that are 14 inches long. What are the possible numbers of strips that Riley can cut?

Selected Response
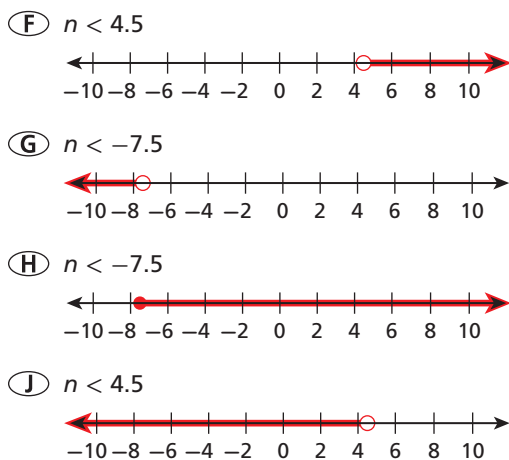
1. Graph the inequality $m < -3.4$.

A

B

C

D

2. Describe the solutions of $6 + y < 10$ in words.

F The value of y is a number less than or equal to 3.

G The value of y is a number less than 4.

H The value of y is a number equal to 3.

J The value of y is a number greater than 4.

3. To join the school swim team, swimmers must be able to swim at least 800 yards without stopping. Let n represent the number of yards a swimmer can swim without stopping. Write an inequality describing which values of n will result in a swimmer making the team.

A $n \leq 800$

C $n > 800$

B $n \geq 800$

D $n < 800$

4. Solve the inequality $n + 6 < -1.5$ and graph the solutions.

F $n < 4.5$

G $n < -7.5$

H $n < -7.5$

J $n < 4.5$

5. Solve the inequality $\frac{z}{-4} \leq 2$.

A $z \geq -8$

C $z \leq 8$

B $z \leq -8$

D $z \geq 8$

6. Carlotta subscribes to the HotBurn music service. She can download no more than 11 song files per week. Carlotta has already downloaded 8 song files this week. Write, solve, and graph an inequality to show how many more songs Carlotta can download.

F $s \leq 3$

G $s > 3$

H $s \geq 3$

J $s < 3$
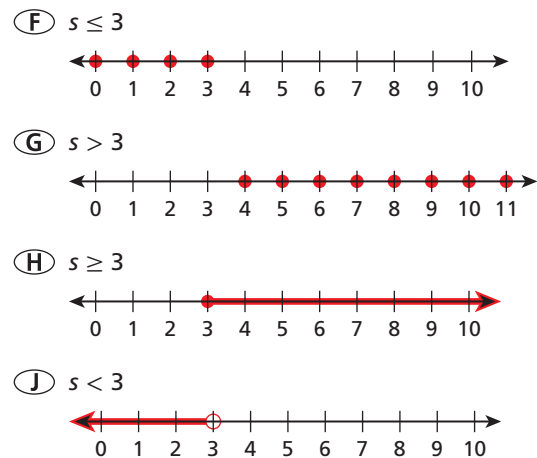
7. Marco's Drama class is performing a play. He wants to buy as many tickets as he can afford. If tickets cost $2.50 each and he has $14.75 to spend, how many tickets can he buy?

A 4 tickets

C 6 tickets

B 0 tickets

D 5 tickets

Mini-Task

8. Glen raised $275 for his softball team's fundraiser. He wants to raise at least $715.

a. Write and solve an inequality to determine how much more money Glen must raise to reach his goal. Let d represent the amount of money in dollars Glen must raise to reach his goal.

b. If Glen raises $50 per week, what is the minimum number of weeks it will take him to reach his goal?